Minkowského prostoročas

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

čtyřvektor

(Pseudo)metrický tenzor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g^{\mu\nu}$$

def. jako inverzní k $g_{\mu\nu}$

interval (invariant)

$$\Delta s^2 = c^2t^2 - x^2 - y^2 - z^2 = g_{\mu\nu} x^\mu x^\nu = x_\nu x^\nu = x^\mu x_\mu$$

$\Downarrow$

Lorentzovy transformace $x'^\mu = a^\mu{}_\nu x^\nu$

(takové lineární transformace které zachovávají interval)

snižování a zvedání indexů

$$x_\mu := g_{\mu\nu} x^\nu$$

$$x^\mu = g^{\mu\nu} x_\nu$$

čtyřrychlost

$$u^\mu := \frac{dx^\mu}{d\tau} = (\gamma c, \gamma \vec{v}) = \left(\frac{c}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad u^\mu u_\mu = c^2$$

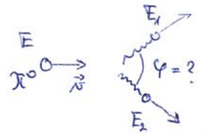
čtyřhybnost

$$p^\mu = m_0 u^\mu = \left(\frac{m_0 c}{\sqrt{\ }}, \frac{m_0 \vec{v}}{\sqrt{\ }}\right) = \left(\frac{E}{c}, \vec{p}\right) \qquad p^\mu p_\mu = m_0^2 c^2$$

vztah mezi energií a hybností $E^2 = m_0^2 c^4 + p^2 c^2$ kde $E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\vec{p} = m \cdot \vec{v} = \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

7.27 Mezon π^0 s klidovou hmotností m_0 pohybující se rychlostí v se rozpadá na dvě stejná kvanta záření gama (fotony). Určete úhel který budou svírat směry pohybu fotonů $\varphi = ?$



v Laboratorní soustavě:

čtyřhybnost mezonu $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$

fotonů $p_i^\mu = \left(\frac{E_i}{c}, \frac{E_i}{c}\vec{s}_i\right) = \left(\frac{E_1}{c}, \frac{E_1}{c}\vec{s}_i\right) \qquad |\vec{s}_i| = 1$

stejná kvanta $\Rightarrow E_1 = E_2$

ZZ energie - hybnosti: $p^\mu = p_1^\mu + p_2^\mu \quad /^2$

$$\underbrace{p^\mu p_\mu}_{m_0^2c^2} = (p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = \underbrace{p_1^\mu p_{1\mu}}_{0} + p_1^\mu p_{2\mu} + \underbrace{p_2^\mu p_{1\mu}}_{p_1^\mu p_{2\mu}} + \underbrace{p_2^\mu p_{2\mu}}_{0}$$

ZZE $\quad E = E_1 + E_2 = 2E_1$

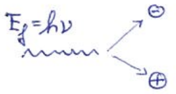
$$\frac{E}{2} = E_1$$

$$m_0^2c^2 = 2p_1^\mu p_{2\mu} = 2p_{1\mu}p_2^\mu = 2\left(\frac{E_1}{c}\frac{E_2}{c} - \frac{E_1}{c}\frac{E_1}{c}\underbrace{\vec{s}_1\vec{s}_2}_{\cos\varphi}\right) = 2\frac{E_1^2}{c^2}(1-\cos\varphi)$$

$$\longrightarrow \quad \frac{m_0^2c^4}{2E_1^2} = (1-\cos\varphi) = 1 - \cos^2\frac{\varphi}{2} + \sin^2\frac{\varphi}{2}$$

$$2\left(\sqrt{1-\frac{v^2}{c^2}}\right)^2 = \frac{2m_0^2c^4}{E^2} = \frac{m_0^2c^4}{\frac{2E^2}{4}} = 2 - 2\cos^2\frac{\varphi}{2} \quad \Rightarrow \quad \frac{v^2}{c^2} = \cos^2\frac{\varphi}{2} \quad \Rightarrow \quad \boxed{\frac{v}{c} = \cos\frac{\varphi}{2}}$$

7.28 Dokažte, že v nepřítomnosti vnějšího pole se foton nemůže změnit v pár positron - elektron.



$$p_f^\mu = \frac{h\nu}{c}(1, \vec{s})$$

$$p_\pm^\mu = \left(\frac{E_\pm}{c}; \vec{p}_\pm\right)$$

$$\frac{E_\pm}{c} = \sqrt{p_\pm^2 + m_{0\pm}^2c^2}$$

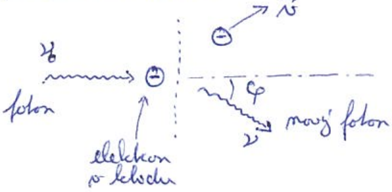
ZZEH: $p_f^\mu = p_+^\mu + p_-^\mu \quad /^2$

$$0 = \underbrace{p_f^\mu p_{f\mu}}_{0} = (p_+^\mu + p_-^\mu)(p_{+\mu} + p_{-\mu}) = \underbrace{p_+^\mu p_{+\mu}}_{m_{0+}^2c^2} + 2p_+^\mu p_{-\mu} + \underbrace{p_-^\mu p_{-\mu}}_{m_{0-}^2c^2}$$

$$0 = m_{0+}^2c^2 + 2\left(\frac{E_+}{c}\frac{E_-}{c} - \vec{p}_+\vec{p}_-\right) + m_{0-}^2c^2 = \underbrace{m_{0+}^2c^2 + m_{0-}^2c^2}_{>0} + 2\underbrace{\left(\sqrt{p_+^2 + m_{0+}^2c^2}\sqrt{p_-^2 + m_{0-}^2c^2} - |\vec{p}_+||\vec{p}_-|\cos\varphi\right)}_{>0} \Rightarrow \text{spor.}$$

Jinak: $E_+ + E_- = E_f = c|\vec{p}_f| = c|\vec{p}_+ + \vec{p}_-| \leq c(|\vec{p}_+| + |\vec{p}_-|) = cp_+ + cp_- < E_+ + E_-$ spor.

Comptonův jev



v laboratorní soustavě

foton $\quad p_{0f}^\mu = \frac{h\nu_0}{c}(1, \vec{s}_0) \qquad p_f^\mu = \frac{h\nu}{c}(1, \vec{s})$

elektron $\quad p_{0e}^\mu = \left(\frac{E_{0e}}{c}, 0\right) = (m_0c, 0) \qquad p_e^\mu = \left(\frac{E_e}{c}, \vec{p}_e\right)$

ZZHE

$$p_{0f}^\mu + p_{0e}^\mu = p_f^\mu + p_e^\mu$$

$$p_{0f}^\mu - p_f^\mu = p_e^\mu - p_{0e}^\mu \quad /^2$$

ZZE: $E_{0f} + E_{0e} = E_f + E_e$

$$E_e = E_{0e} + E_{0f} - E_f$$

$$E_e = E_{0e} + h(\nu_0 - \nu)$$

Jinak ZZE:

$$h\nu_0 + E_{0e} = h\nu + E_e$$

ZZH:

$$\frac{h\nu_0}{c}\vec{s}_0 + \underbrace{\vec{p}_{e0}}_{0} = \frac{h\nu}{c}\vec{s} + \vec{p}_e$$

$$\underbrace{p_{0f}^\mu p_{0f\mu}}_{0} - 2p_{0f}^\mu p_{f\mu} + \underbrace{p_f^\mu p_{f\mu}}_{0} = \underbrace{p_e^\mu p_{e\mu}}_{} - 2p_e^\mu p_{0e\mu} + \underbrace{p_{0e}^\mu p_{0e\mu}}_{m_0^2c^2}$$

$$0 - 2\frac{h\nu}{c}\frac{h\nu_0}{c}(1 - \vec{s}_0\vec{s}) + 0 = m_0^2c^2 - 2\left(\frac{E_e}{c}m_0c\right) + m_0^2c^2 = 2m_0^2c^2 - 2\left(m_0E_{0e} + m_0h(\nu_0 - \nu)\right)$$

$$+2\frac{h^2}{c^2}\nu\nu_0(1 - \underbrace{\vec{s}_0\vec{s}}_{\cos\varphi}) = +2m_0h(\nu_0 - \nu)$$

$$\nu\left(\frac{h^2}{c^2}\nu_0(1-\cos\varphi) + m_0h\right) = m_0h\nu_0$$

$$\nu = \frac{m_0h\nu_0}{m_0h + \frac{h^2}{c^2}\nu_0\underbrace{(1-\cos\varphi)}_{=1-\cos^2\frac{\varphi}{2}+\sin^2\frac{\varphi}{2} = 2\sin^2\frac{\varphi}{2}}} = \frac{\nu_0}{1 + \frac{h}{m_0c^2}\nu_0(1-\cos\varphi)} = \frac{\nu_0}{1 + \frac{2h\nu_0}{m_0c^2}\sin^2\left(\frac{\varphi}{2}\right)}$$