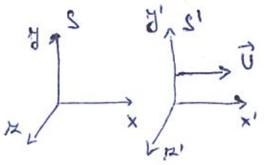
STR

Speciální Lorentzova Transformace



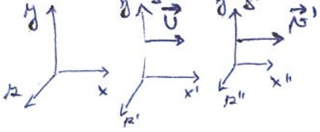
$$x' = \frac{x - Ut}{\sqrt{1-\frac{U^2}{c^2}}} \qquad y' = y \qquad z' = z \qquad \beta = \frac{U}{c}$$

$$t' = \frac{t - \frac{U}{c^2}x}{\sqrt{1-\frac{U^2}{c^2}}} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

maticově

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \qquad x'^\mu = \alpha^\mu{}_\nu x^\nu \qquad \alpha^\mu{}_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

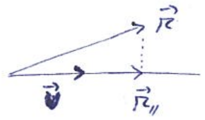
7.3 Odvoďte zákon skládání rovnoběžných rychlostí složením dvou speciálních Lorentzových transformací. Jsou tyto transformace záměnné? (záleží na jejich pořadí - komutují jejich matice?).



$$x' = \frac{x - Ut}{\sqrt{1-\frac{U^2}{c^2}}} \qquad t' = \frac{t - \frac{U}{c^2}x}{\sqrt{1-\frac{U^2}{c^2}}}$$

$$x'' = \frac{x' - v't'}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{\frac{x-Ut}{\sqrt{1-\frac{U^2}{c^2}}} - v'\frac{t - \frac{U}{c^2}x}{\sqrt{1-\frac{U^2}{c^2}}}}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{x\left(1+\frac{Uv'}{c^2}\right) - (U+v')t}{\sqrt{1-\frac{v'^2}{c^2}}\sqrt{1-\frac{U^2}{c^2}}} = \frac{x - \overbrace{\frac{U+v'}{\left(1+\frac{Uv'}{c^2}\right)}}^{v \;\ldots\; \text{rychlost } S'' \text{ vůči } S} t}{\sqrt{\frac{1-\frac{v'^2}{c^2}-\frac{U^2}{c^2}+\frac{U^2v'^2}{c^4}+\frac{2Uv'}{c^2}-\frac{2Uv'}{c^2}}{1+\frac{2Uv'}{c^2}+\frac{U^2v'^2}{c^4}}}} = \frac{x - \frac{U+v'}{\left(1+\frac{Uv'}{c^2}\right)}t}{\sqrt{1 - \frac{1}{c^2}\frac{(U+v')^2}{\left(1+\frac{Uv'}{c^2}\right)^2}}}$$

7.2 Lorentzova transformace vektorově - speciální ve směru $\frac{\vec{U}}{|\vec{U}|}$. Víme $x' = \gamma(x - Ut)$ $y' = y$ $z' = z$ $t' = \gamma(t - \frac{U}{c^2}x)$ transformuje se jen pomocí podélné složky, tj $\vec{r}_\parallel$



$$\vec{r}_\parallel = \frac{\vec{r}\cdot\vec{U}}{U}\cdot\frac{\vec{U}}{U} = \frac{\vec{r}\cdot\vec{U}}{U^2}\vec{U} \qquad \vec{r}_\perp = \vec{r} - \vec{r}_\parallel$$

$$\vec{r}' = \vec{r}'_\parallel + \vec{r}'_\perp = \gamma(\vec{r}_\parallel - \vec{U}t) + \vec{r} - \vec{r}_\parallel = (\gamma-1)\vec{r}_\parallel - \gamma\vec{U}t + \vec{r} = \vec{r} + (\gamma-1)\frac{\vec{r}\vec{U}}{U^2}\vec{U} - \gamma t\vec{U} = \underline{\underline{\vec{r} + \left(\frac{(\gamma-1)}{U^2}(\vec{r}\cdot\vec{U}) - \gamma t\right)\vec{U}}}$$

$$t' = \gamma\left(t - \frac{\vec{U}}{c^2}\vec{r}_\parallel\right) = \underline{\underline{\gamma\left(t - \frac{\vec{U}}{c^2}\vec{r}\right)}}$$

7.1 Najděte matici pro transformaci z 7.2. - Boost ve směru $\vec{U}$ ... speciální Lorentzova transformace ve směru $\vec{U}$

$$(*) \quad t' = \gamma\left(t - \frac{\vec{U}}{c^2}\vec{r}\right) \qquad \vec{r}' = \vec{r} + \left(\frac{(\gamma-1)}{U^2}(\vec{r}\cdot\vec{U}) - \gamma t\right)\vec{U}$$

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \qquad \text{báze } \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$x'^0 = ct' = \gamma\Big(\underbrace{ct}_{x^0} - \frac{\vec{U}\vec{r}}{c}\Big)$$

$$\vec{r}' = \vec{r} + \Big(\frac{\gamma-1}{U^2}\vec{r}\cdot\vec{U} - \frac{\gamma}{c}\underbrace{ct}_{x^0}\Big)\vec{U}$$

matice zobrazení v bazích $\mathcal{E}, \tilde{\mathcal{E}}$

$${}_{\mathcal{E}}A^{\tilde{\mathcal{E}}} = \left([A e_1]_{\tilde{\mathcal{E}}}, \ldots, [A e_n]_{\tilde{\mathcal{E}}}\right)$$

$$A = \begin{pmatrix} \gamma & -\frac{\gamma}{c}U_1 & -\frac{\gamma}{c}U_2 & -\frac{\gamma}{c}U_3 \\ -\frac{\gamma}{c}U_1 & 1+\frac{\gamma-1}{U^2}U_1U_1 & \frac{\gamma-1}{U^2}U_2U_1 & \frac{\gamma-1}{U^2}U_3U_1 \\ -\frac{\gamma}{c}U_2 & \frac{\gamma-1}{U^2}U_1U_2 & 1+\frac{\gamma-1}{U^2}U_2U_2 & \frac{\gamma-1}{U^2}U_3U_2 \\ -\frac{\gamma}{c}U_3 & \frac{\gamma-1}{U^2}U_1U_3 & \frac{\gamma-1}{U^2}U_2U_3 & 1+\frac{\gamma-1}{U^2}U_3U_3 \end{pmatrix}$$

7.4 Odvoďte zákon skládání rychlostí pro libovolnou vzájemnou orientaci rychlostí.

invertujeme vztahy (*)

$$t = \gamma'\left(t' - \frac{\vec{U}'}{c^2}\vec{r}'\right) = \gamma\left(t' + \frac{\vec{U}\vec{r}'}{c^2}\right) \qquad \vec{r} = \vec{r}(t) = \vec{r}(\vec{r}'(t'), t')$$

$$\vec{r} = \vec{r}' + \left(\frac{\gamma-1}{U^2}\vec{U}\vec{r}' + \gamma t'\right)\vec{U}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dt'}\cdot\frac{dt'}{dt} = \frac{d\vec{r}}{dt'}\frac{1}{\frac{dt}{dt'}} = \left[\overbrace{\frac{d\vec{r}'}{dt'}}^{\vec{v}'} + \left(\frac{\gamma-1}{U^2}\vec{U}\frac{d\vec{r}'}{dt'} + \gamma\right)\vec{U}\right]\frac{1}{\gamma\left(1+\frac{\vec{U}\vec{v}'}{c^2}\right)} = \frac{\vec{v}' + \left(\frac{\gamma-1}{U^2}\vec{U}\vec{v}' + \gamma\right)\vec{U}}{\gamma\left(1+\frac{\vec{U}\vec{v}'}{c^2}\right)} = \underline{\underline{\frac{\vec{U} + \vec{v}'\sqrt{1-\frac{U^2}{c^2}} + \left(1 - \sqrt{1-\frac{U^2}{c^2}}\right)\frac{\vec{U}\vec{v}'}{U^2}\vec{U}}{1+\frac{\vec{U}\vec{v}'}{c^2}}}}$$