Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r-holonomních (nezávislých)

$f_k(\vec{x}, t) = 0 \quad \forall k \in \hat{r}$ $\qquad h(\underbrace{\nabla f_1, \dots, \nabla f_r}_{\text{L.N.}}) = r \quad \forall \vec{x} \in M(t)$

$0 = \delta f_k = \frac{\partial f_k}{\partial x_i} \delta x_i = \nabla f_k \cdot \delta \vec{x}$

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r−holonomních (nezávislých)   p−neholonomních (nezávislých)

$f_k(\vec{x},t)=0 \quad \forall k\in\hat{r}$ $h(\underbrace{\nabla f_1,\dots,\nabla f_r}_{\text{L.N.}})=r \quad \forall \vec{x}\in M(t)$

$0=\delta f_k=\frac{\partial f_k}{\partial x_i}\delta x_i=\nabla f_k\cdot\delta\vec{x}$

$\sum_{i=1}^{3N} a_{li}(\vec{x},t)\dot{x}_i+b_l(\vec{x},t)=0 \quad \forall l\in\hat{p}$ $\mathbb{A}=(a_{li})$ matice typu $p\times 3N$

$\mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0$ $h(\mathbb{A}(\vec{x},t))=p \quad \forall \vec{x}\in M(t)\ \forall t$

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r – holonomních (nezávislých) $\qquad$ p – neholonomních (nezávislých)

$f_k(\vec{x},t)=0 \quad \forall k\in\hat{r}$ $\qquad$ $h(\underbrace{\nabla f_1,\dots,\nabla f_r}_{\text{L.N.}\quad \forall \vec{x}\in M(t)})=r$

$0=\delta f_k=\frac{\partial f_k}{\partial x_i}\delta x_i=\nabla f_k\cdot\delta\vec{x}$

$\sum_{i=1}^{3N} a_{li}(\vec{x},t)\dot{x}_i+b_l(\vec{x},t)=0 \quad \forall l\in\hat{p}$ $\qquad$ $\mathbb{A}=(a_{li})$ matice typu $p\times 3N$

$\mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \quad /\cdot dt$ $\qquad$ $h(\mathbb{A}(\vec{x},t))=p \quad \forall\vec{x}\in M(t)\ \forall t$

$\mathbb{A}(\vec{x},t)\,d\vec{x}+\vec{b}(\vec{x},t)\,dt=0 \leftarrow \dot{\vec{x}}\,dt=d\vec{x}$

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r-holonomních (nezávislých)

$f_k(\vec{x},t)=0 \quad \forall k\in\hat{r}$ $\qquad h(\underbrace{\nabla f_1,\dots,\nabla f_r}_{\text{L.N.}})=r \quad \forall \vec{x}\in M(t)$

$0=\delta f_k=\frac{\partial f_k}{\partial x_i}\delta x_i=\nabla f_k\cdot\delta\vec{x}$

p-neholonomních (nezávislých)

$\sum_{i=1}^{3N} a_{li}(\vec{x},t)\dot{x}_i+b_l(\vec{x},t)=0 \quad \forall l\in\hat{p}$ $\qquad \mathbb{A}=(a_{li})$ matice typu $p\times 3N$

$\mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \quad /\cdot dt$ $\qquad h(\mathbb{A}(\vec{x},t))=p \quad \forall\vec{x}\in M(t)\ \forall t$

$\mathbb{A}(\vec{x},t)d\vec{x}+\vec{b}(\vec{x},t)\,dt=0 \leftarrow \dot{\vec{x}}\,dt=d\vec{x} \quad / \ dt=0 \Rightarrow d\vec{x}\to\delta\vec{x}$

$\mathbb{A}(\vec{x},t)\delta\vec{x}=0 \qquad \sum_{i=1}^{3N} a_{li}(\vec{x},t)\delta x_i=0 \quad \forall l\in\hat{p}$

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r-holonomních (nezávislých)

$$f_k(\vec{x},t)=0 \quad \forall k \in \hat{r} \qquad h(\underbrace{\nabla f_1,\dots,\nabla f_r}_{\text{L.N.} \quad \forall \vec{x}\in M(t)})=r$$

$$0=\delta f_k=\frac{\partial f_k}{\partial x_i}\delta x_i=\nabla f_k\cdot\delta\vec{x}$$

p-neholonomních (nezávislých)

$$\sum_{i=1}^{3N} a_{li}(\vec{x},t)\dot{x}_i+b_l(\vec{x},t)=0 \quad \forall l\in\hat{p} \qquad \mathbb{A}=(a_{li}) \text{ matice typu } p\times 3N$$

$$\mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \quad /\cdot dt \qquad h(\mathbb{A}(\vec{x},t))=p \quad \forall\vec{x}\in M(t)\ \forall t$$

$$\mathbb{A}(\vec{x},t)d\vec{x}+\vec{b}(\vec{x},t)\,dt=0 \leftarrow \dot{\vec{x}}dt=d\vec{x} \quad /\ dt=0 \Rightarrow d\vec{x}\to\delta\vec{x}$$

$$\mathbb{A}(\vec{x},t)\delta\vec{x}=0 \qquad \sum_{i=1}^{3N} a_{li}(\vec{x},t)\delta x_i=0 \quad \forall l\in\hat{p}$$

d'Alembertův princip

$$\delta A_{ef}=\sum_{i=1}^{3N}(F_i-m_i\ddot{x}_i)\cdot\delta x_i=(\vec{F}-\mathbb{M}\ddot{\vec{x}})\cdot\delta\vec{x}=0 \qquad (1)$$

$$\left.\begin{array}{l} f_k(\vec{x},t)=0 \quad \forall k\in\hat{r} \\ \mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \end{array}\right\}(3) \qquad \forall\delta\vec{x}\in\mathbb{R}^{3N} \quad \left.\begin{array}{r} \nabla f_k\cdot\delta\vec{x}=0 \\ \mathbb{A}\delta\vec{x}=0 \end{array}\right\}(2)$$

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r−holonomních (nezávislých) p−neholonomních (nezávislých)

$f_k(\vec{x},t)=0 \quad \forall k\in\hat{r}$ $\qquad h(\underbrace{\nabla f_1,\dots,\nabla f_r}_{\text{L.N.}\quad \forall\vec{x}\in M(t)})=r$

$0=\delta f_k=\frac{\partial f_k}{\partial x_i}\delta x_i=\nabla f_k\cdot\delta\vec{x}$

$\sum_{i=1}^{3N} a_{li}(\vec{x},t)\dot{x}_i+b_l(\vec{x},t)=0 \quad \forall l\in\hat{p}$ $\qquad \mathbb{A}=(a_{li})$ matice typu $p\times 3N$

$\mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \quad /\cdot dt$ $\qquad h(\mathbb{A}(\vec{x},t))=p \quad \forall\vec{x}\in M(t)\ \forall t$

$\mathbb{A}(\vec{x},t)d\vec{x}+\vec{b}(\vec{x},t)\,dt=0 \leftarrow \dot{\vec{x}}dt=d\vec{x} \quad / \ dt=0\Rightarrow d\vec{x}\to\delta\vec{x}$

$\mathbb{A}(\vec{x},t)\delta\vec{x}=0 \qquad \sum_{i=1}^{3N} a_{li}(\vec{x},t)\delta x_i=0 \quad \forall l\in\hat{p}$

d'Alembertův princip

$$\delta A_{ef}=\sum_{i=1}^{3N}(F_i-m_i\ddot{x}_i)\cdot\delta x_i=(\vec{F}-\mathbb{M}\ddot{\vec{x}})\cdot\delta\vec{x}=0 \qquad (1)$$

$$\left.\begin{aligned} f_k(\vec{x},t)&=0 \quad \forall k\in\hat{r}\\ \mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)&=0\end{aligned}\right\}(3) \qquad \forall\delta\vec{x}\in\mathbb{R}^{3N} \quad \left.\begin{aligned}\nabla f_k\cdot\delta\vec{x}&=0\\ \mathbb{A}\delta\vec{x}&=0\end{aligned}\right\}(2)$$

$$\forall\delta\vec{x}\in T_{\vec{x}}M(t)\cap \mathrm{Ker}\,\mathbb{A}(\vec{x},t)=\mathrm{Ker}\,\mathbb{B}(\vec{x},t)$$

$\mathbb{B}(\vec{x},t)$ matice $(r+p)\times 3N$ hodnosti $r+p$

$s=3N-r$

Pohybové rovnice (LR1D) pro neholonomní soustavu s vazbami lineárně závislými na rychlostech

vazby: r–holonomních (nezávislých)

$f_k(\vec{x},t)=0 \quad \forall k\in\hat{r}$ $\qquad h(\underbrace{\nabla f_1,\dots,\nabla f_r}_{\text{L.N.}})=r \quad \forall \vec{x}\in M(t)$

$0=\delta f_k=\frac{\partial f_k}{\partial x_i}\delta x_i=\nabla f_k\cdot\delta\vec{x}$

d'Alembertův princip

$$\delta A_{ef}=\sum_{i=1}^{3N}(F_i-m_i\ddot{x}_i)\cdot\delta x_i=(\vec{F}-\mathbb{M}\ddot{\vec{x}})\cdot\delta\vec{x}=0 \qquad (1)$$

$$\left.\begin{array}{l} f_k(\vec{x},t)=0 \quad \forall k\in\hat{r} \\ \mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \end{array}\right\}(3) \qquad \forall\delta\vec{x}\in\mathbb{R}^{3N} \quad \left.\begin{array}{r} \nabla f_k\cdot\delta\vec{x}=0 \\ \mathbb{A}\delta\vec{x}=0 \end{array}\right\}(2)$$

$$\forall\delta\vec{x}\in T_{\vec{x}}M(t)\cap\operatorname{Ker}\mathbb{A}(\vec{x},t)=\operatorname{Ker}\mathbb{B}(\vec{x},t)$$

$\mathbb{B}(\vec{x},t)$ matice $(r+p)\times 3N$ hodnosti $r+p$

$$s=3N-r$$

p–neholonomních (nezávislých)

$$\sum_{i=1}^{3N}a_{li}(\vec{x},t)\dot{x}_i+b_l(\vec{x},t)=0 \quad \forall l\in\hat{p}$$

$\mathbb{A}=(a_{li})$ matice typu $p\times 3N$

$$\mathbb{A}(\vec{x},t)\dot{\vec{x}}+\vec{b}(\vec{x},t)=0 \quad /\cdot dt$$

$h(\mathbb{A}(\vec{x},t))=p \quad \forall\vec{x}\in M(t)\ \forall t$

$$\mathbb{A}(\vec{x},t)d\vec{x}+\vec{b}(\vec{x},t)\,dt=0 \leftarrow \dot{\vec{x}}dt=d\vec{x} \quad / \ dt=0 \Rightarrow d\vec{x}\to\delta\vec{x}$$

$$\mathbb{A}(\vec{x},t)\delta\vec{x}=0 \qquad \sum_{i=1}^{3N}a_{li}(\vec{x},t)\delta x_i=0 \quad \forall l\in\hat{p}$$

Metoda Lagrangeových multiplikátorů:
Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j,μ_j (funkce $(\vec{x},\dot{\vec{x}},\ddot{\vec{x}},t)$) a přičteme k rovnici (1) a nastavíme λ_j,μ_j tak abychom vynulovali koeficienty u závislých δx_j

Metoda Lagrangeových multiplikátorů:

Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

$s = 3N - r$

$$\underbrace{\mathbb{B}(\vec{x}, t)}_{}\ \text{matice } (r+p)\times 3N \text{ hodnosti } r+p$$

$$\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{s-p+1}} & \cdots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial x_1} & \cdots & \frac{\partial f_r}{\partial x_{s-p+1}} & \cdots & \frac{\partial f_r}{\partial x_{3N}} \\ a_{1,1} & \cdots & a_{1,s-p+1} & \cdots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \cdots & a_{p,s-p+1} & \cdots & a_{p,3N} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} = 0$$

Metoda Lagrangeových multiplikátorů:

Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

$$s = 3N - r$$

$\mathbb{B}(\vec{x}, t)$ matice $(r+p)\times 3N$ hodnosti $r+p$

BÚNO: regulární matice typu $(r+p)\times(r+p)$

$$\underbrace{\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{s-p+1}} & \cdots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial x_1} & \cdots & \frac{\partial f_r}{\partial x_{s-p+1}} & \cdots & \frac{\partial f_r}{\partial x_{3N}} \\ a_{1,1} & \cdots & a_{1,s-p+1} & \cdots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \cdots & a_{p,s-p+1} & \cdots & a_{p,3N} \end{pmatrix}}_{} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{nezávislé} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{závislé} \end{matrix} = 0$$

Metoda Lagrangeových multiplikátorů:

Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

$$s = 3N - r$$

$$(\lambda_1, \dots, \lambda_r, \mu_1, \dots, \mu_p) \overbrace{\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{s-p+1}} & \cdots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial x_1} & \cdots & \frac{\partial f_r}{\partial x_{s-p+1}} & \cdots & \frac{\partial f_r}{\partial x_{3N}} \\ a_{1,1} & \cdots & a_{1,s-p+1} & \cdots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \cdots & a_{p,s-p+1} & \cdots & a_{p,3N} \end{pmatrix}}^{\mathbb{B}(\vec{x},t) \text{ matice } (r+p)\times 3N \text{ hodnosti } r+p;\ \text{BÚNO: regulární matice typu } (r+p)\times(r+p) \text{ (sloupce } s-p+1,\dots,3N)} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{nezávislé} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{závislé} \end{array} = 0$$

$$0 = \sum_{i=1}^{3N} \left(F_i - m_i \ddot{x}_i + \sum_{k=1}^{r} \lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p} \mu_l a_{li} \right) \underbrace{\delta x_i}_{\delta x_1, \dots, \delta x_{3N} \text{ nejsou nezávislé}}$$

Metoda Lagrangeových multiplikátorů:

Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

$$s = 3N - \pi$$

$\mathbb{B}(\vec{x}, t)$ matice $(\pi+p)\times 3N$ hodnosti $\pi+p$

BÚNO: regulární matice typu $(\pi+p)\times(\pi+p)$

$$(\lambda_1,\dots,\lambda_\pi,\mu_1,\dots,\mu_p)\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_{s-p+1}} & \dots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_\pi}{\partial x_1} & \dots & \frac{\partial f_\pi}{\partial x_{s-p+1}} & \dots & \frac{\partial f_\pi}{\partial x_{3N}} \\ a_{1,1} & \dots & a_{1,s-p+1} & \dots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \dots & a_{p,s-p+1} & \dots & a_{p,3N} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} \begin{array}{l} \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} \text{nezávislé} \\ \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} \text{závislé} \end{array} = 0$$

$$(-F_1^{ef}, \dots, -F_{s-p+1}^{ef}, \dots, -F_{3N}^{ef})$$

$$0 = \sum_{i=1}^{3N}\left(F_i - m_i\ddot{x}_i + \sum_{k=1}^{\pi}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\right)\underbrace{\delta x_i}_{} = \sum_{i=1}^{s-p}\left(F_i - m_i\ddot{x}_i + \sum_{k=1}^{\pi}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\right)\delta x_i + \sum_{i=s-p+1}^{3N}\left(F_i - m_i\ddot{x}_i + \sum_{k=1}^{\pi}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\right)\delta x_i$$

$\delta x_1, \dots, \delta x_{3N}$ nejsou nezávislé

Metoda Lagrangeových multiplikátorů:
Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

funkce $(\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}, t)$

$s = 3N - r$

$\mathbb{B}(\vec{x}, t)$ matice $(r+p)\times 3N$ hodnosti $r+p$

BÚNO: regulární matice typu $(r+p)\times(r+p)$

$$\underbrace{(\lambda_1,\dots,\lambda_r,\mu_1,\dots,\mu_p)}_{\text{nastavíme tak aby}}
\begin{pmatrix}
\frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_{s-p+1}} & \dots & \frac{\partial f_1}{\partial x_{3N}}\\
\vdots & & \vdots & & \vdots\\
\frac{\partial f_r}{\partial x_1} & \dots & \frac{\partial f_r}{\partial x_{s-p+1}} & \dots & \frac{\partial f_r}{\partial x_{3N}}\\
a_{1,1} & \dots & a_{1,s-p+1} & \dots & a_{1,3N}\\
\vdots & & \vdots & & \vdots\\
a_{p,1} & \dots & a_{p,s-p+1} & \dots & a_{p,3N}
\end{pmatrix}
\begin{pmatrix}
\delta x_1\\ \vdots \\ \delta x_{s-p}\\ \delta x_{s-p+1}\\ \vdots \\ \delta x_{3N}
\end{pmatrix} = 0$$

$\delta x_1, \dots, \delta x_{s-p}$ nezávislé; $\delta x_{s-p+1}, \dots, \delta x_{3N}$ závislé

$$\begin{matrix} * & & = & & = \\ (-F_1^{ef}, & \dots, & -F_{s-p+1}^{ef}, & \dots, & -F_{3N}^{ef}) \end{matrix}$$

$$0 = \sum_{i=1}^{3N}\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)\underbrace{\delta x_i}_{\delta x_1,\dots,\delta x_{3N}\text{ nejsou nezávislé}} = \sum_{i=1}^{s-p}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{0\ \Leftarrow}\underbrace{\delta x_i}_{\delta x_1,\dots,\delta x_{s-p}\text{ jsou nezávislé}} + \sum_{i=s-p+1}^{3N}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{0\text{ vynulujeme volbou }\lambda_k\text{ a }\mu_l}\delta x_i$$

Metoda Lagrangeových multiplikátorů:

Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

$s = 3N - r$

$\mathbb{B}(\vec{x}, t)$ matice $(r+p) \times 3N$ hodnosti $r+p$

BÚNO: regulární matice typu $(r+p)\times(r+p)$

$$\underbrace{(\lambda_1, \dots, \lambda_r, \mu_1, \dots, \mu_p)}_{\text{nastavíme tak aby}} \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_{s-p+1}} & \dots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial x_1} & \dots & \frac{\partial f_r}{\partial x_{s-p+1}} & \dots & \frac{\partial f_r}{\partial x_{3N}} \\ a_{1,1} & \dots & a_{1,s-p+1} & \dots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \dots & a_{p,s-p+1} & \dots & a_{p,3N} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} = 0$$

$\delta x_1, \dots, \delta x_{s-p}$ nezávislé; $\delta x_{s-p+1}, \dots, \delta x_{3N}$ závislé

$(-F_1^{ef}, \dots, -F_{s-p+1}^{ef}, \dots, -F_{3N}^{ef})$

funkce $(\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}, t)$

$$0 = \sum_{i=1}^{3N}\left(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\right)\underbrace{\delta x_i}_{\delta x_1, \dots, \delta x_{3N} \text{ nejsou nezávislé}} = \sum_{i=1}^{s-p}\underbrace{\left(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\right)}_{0 \;\Leftarrow\; \delta x_1, \dots, \delta x_{s-p} \text{ jsou nezávislé}}\delta x_i + \sum_{i=s-p+1}^{3N}\underbrace{\left(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\right)}_{0 \text{ vynulujeme volbou } \lambda_k \text{ a } \mu_l}\delta x_i$$

Pohybové rovnice: $m_i\ddot{x}_i = F_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li} \quad \forall i \in \widehat{3N}$ $\quad f_k(\vec{x}, t) = 0 \;\forall k \in \hat{r}$ $\quad \sum_{j=1}^{3N} a_{lj}(\vec{x}, t)\dot{x}_j + b_l(\vec{x}, t) = 0 \quad \forall l \in \hat{p}$

Metoda Lagrangeových multiplikátorů:

Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

funkce $(\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}, t)$

$s = 3N - r$

$\underbrace{(\lambda_1, \dots, \lambda_r, \mu_1, \dots, \mu_p)}_{\text{nastavíme tak aby}}$

$$\overbrace{\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_{s-p+1}} & \dots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial x_1} & \dots & \frac{\partial f_r}{\partial x_{s-p+1}} & \dots & \frac{\partial f_r}{\partial x_{3N}} \\ a_{1,1} & \dots & a_{1,s-p+1} & \dots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \dots & a_{p,s-p+1} & \dots & a_{p,3N} \end{pmatrix}}^{\mathbb{B}(\vec{x},t) \text{ matice } (r+p)\times 3N \text{ hodnosti } r+p;\ \text{BÚNO: regulární matice typu } (r+p)\times(r+p)} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} \begin{array}{l} \left.\vphantom{\begin{matrix} a \\ b \\ c \end{matrix}}\right\} \text{nezávislé} \\ \left.\vphantom{\begin{matrix} a \\ b \\ c \end{matrix}}\right\} \text{závislé} \end{array} = 0$$

$$* \qquad \| \qquad \|$$

$$(-F_1^{ef}, \dots, -F_{s-p+1}^{ef}, \dots, -F_{3N}^{ef})$$

$$0 = \sum_{i=1}^{3N}\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)\underbrace{\delta x_i}_{\delta x_1, \dots, \delta x_{3N} \text{ nejsou nezávislé}} = \sum_{i=1}^{s-p}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{0 \text{ " } \Leftarrow}\underbrace{\delta x_i}_{\delta x_1, \dots, \delta x_{s-p} \text{ jsou nezávislé}} + \sum_{i=s-p+1}^{3N}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{0 \text{ " vynulujeme volbou } \lambda_k \text{ a } \mu_l}\delta x_i$$

Pohybové rovnice: $m_i\ddot{x}_i = F_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li} \quad \forall i \in \widehat{3N}$ $\qquad f_k(\vec{x}, t) = 0 \ \forall k \in \hat{r}$ $\qquad \sum_{j=1}^{3N} a_{lj}(\vec{x}, t)\dot{x}_j + b_l(\vec{x}, t) = 0 \quad \forall l \in \hat{p}$

Věta o energii: $\quad / \dot{x}_i \quad / \sum_{i=1}^{3N}$

$$m_i\ddot{x}_i\dot{x}_i = F_i\dot{x}_i + \sum_{k=1}^{r}\lambda_k \frac{\partial f_k}{\partial x_i}\dot{x}_i + \sum_{l=1}^{p}\mu_l a_{li}\dot{x}_i$$

Metoda Lagrangeových multiplikátorů:
Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

funkce $(\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}, t)$

$\mathbb{B}(\vec{x}, t)$ matice $(r+p)\times 3N$ hodnosti $r+p$

$s = 3N - r$

BÚNO: regulární matice typu $(r+p)\times(r+p)$

$\underbrace{(\lambda_1, \dots, \lambda_r, \mu_1, \dots, \mu_p)}_{\text{nastavíme tak aby}}$

$$
\begin{pmatrix}
\frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_{s-p+1}} & \dots & \frac{\partial f_1}{\partial x_{3N}} \\
\vdots & & \vdots & & \vdots \\
\frac{\partial f_r}{\partial x_1} & \dots & \frac{\partial f_r}{\partial x_{s-p+1}} & \dots & \frac{\partial f_r}{\partial x_{3N}} \\
a_{1,1} & \dots & a_{1,s-p+1} & \dots & a_{1,3N} \\
\vdots & & \vdots & & \vdots \\
a_{p,1} & \dots & a_{p,s-p+1} & \dots & a_{p,3N}
\end{pmatrix}
\begin{pmatrix}
\delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N}
\end{pmatrix} = 0
$$

$\delta x_1, \dots, \delta x_{s-p}$ nezávislé; $\delta x_{s-p+1}, \dots, \delta x_{3N}$ závislé

$$* \qquad \parallel \qquad \parallel$$
$$(-F_1^{ef}, \dots, -F_{s-p+1}^{ef}, \dots, -F_{3N}^{ef})$$

$$0 = \sum_{i=1}^{3N}\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)\delta x_i = \sum_{i=1}^{s-p}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{\text{"}0\text{"} \Leftarrow \delta x_1, \dots, \delta x_{s-p} \text{ jsou nezávislé}}\delta x_i + \sum_{i=s-p+1}^{3N}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{\text{"}0\text{" vynulujeme volbou } \lambda_k \text{ a } \mu_l}\delta x_i$$

$\delta x_1, \dots, \delta x_{3N}$ nejsou nezávislé

Pohybové rovnice: $m_i\ddot{x}_i = F_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li} \quad \forall i \in \widehat{3N} \qquad f_k(\vec{x}, t) = 0 \ \forall k \in \hat{r} \qquad \sum_{j=1}^{3N} a_{lj}(\vec{x}, t)\dot{x}_j + b_l(\vec{x}, t) = 0 \quad \forall l \in \hat{p}$

Věta o energii: $\quad / \dot{x}_i \ / \sum_{i=1}^{3N}$

$$\frac{\hat{d} f_k}{dt} = \frac{\partial f_k}{\partial x_i}\dot{x}_i + \frac{\partial f_k}{\partial t} = 0$$

$$m_i\ddot{x}_i\dot{x}_i = F_i\dot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i}\dot{x}_i + \sum_{l=1}^{p}\mu_l a_{li}\dot{x}_i \quad \Rightarrow \quad \frac{\hat{d}T}{dt} = \vec{F}\cdot\dot{\vec{x}} - \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial t} - \sum_{l=1}^{p}\mu_l b_l$$

Metoda Lagrangeových multiplikátorů:
Místo vyjadření závislých posunutí složek δx_j pomocí $s-p$ nezávislých z podmínek (2) a jejich dosazení do rovnice (1) přenásobíme rovnice (2) Lagrangeovými multiplikátory λ_j, μ_j a přičteme k rovnici (1) a nastavíme λ_j, μ_j tak abychom vynulovali koeficienty u závislých δx_j

funkce $(\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}, t)$

$s = 3N - r$

$\mathbb{B}(\vec{x}, t)$ matice $(r+p) \times 3N$ hodnosti $r+p$

BÚNO: regulární matice typu $(r+p)\times(r+p)$

$\underbrace{(\lambda_1, \dots, \lambda_r, \mu_1, \dots, \mu_p)}_{\text{nastavíme tak aby}}$

$$(\lambda_1, \dots, \lambda_r, \mu_1, \dots, \mu_p)\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_{s-p+1}} & \dots & \frac{\partial f_1}{\partial x_{3N}} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial f_r}{\partial x_1} & \dots & \frac{\partial f_r}{\partial x_{s-p+1}} & \dots & \frac{\partial f_r}{\partial x_{3N}} \\ a_{1,1} & \dots & a_{1,s-p+1} & \dots & a_{1,3N} \\ \vdots & & \vdots & & \vdots \\ a_{p,1} & \dots & a_{p,s-p+1} & \dots & a_{p,3N} \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \vdots \\ \delta x_{s-p} \\ \delta x_{s-p+1} \\ \vdots \\ \delta x_{3N} \end{pmatrix} = 0$$

$\delta x_1, \dots, \delta x_{s-p}$ nezávislé; $\delta x_{s-p+1}, \dots, \delta x_{3N}$ závislé

$$(-F_1^{ef}, \dots, -F_{s-p+1}^{ef}, \dots, -F_{3N}^{ef})$$

$$0 = \sum_{i=1}^{3N}\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)\underbrace{\delta x_i}_{} = \sum_{i=1}^{s-p}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{}\underbrace{\delta x_i}_{} + \sum_{i=s-p+1}^{3N}\underbrace{\Big(F_i - m_i\ddot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li}\Big)}_{}\delta x_i$$

$\delta x_1, \dots, \delta x_{3N}$ nejsou nezávislé $\quad$ $0'' \Leftarrow \delta x_1, \dots, \delta x_{s-p}$ jsou nezávislé $\quad$ $0''$ vynulujeme volbou λ_k a μ_l

Pohybové rovnice: $m_i\ddot{x}_i = F_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i} + \sum_{l=1}^{p}\mu_l a_{li} \quad \forall i \in \widehat{3N} \qquad f_k(\vec{x}, t) = 0 \ \forall k \in \hat{r} \qquad \sum_{j=1}^{3N} a_{lj}(\vec{x}, t)\dot{x}_j + b_l(\vec{x}, t) = 0 \quad \forall l \in \hat{p}$

Věta o energii: $\quad / \dot{x}_i \ / \sum_{i=1}^{3N}$

$$\frac{\hat{d} f_k}{dt} = \frac{\partial f_k}{\partial x_i}\dot{x}_i + \frac{\partial f_k}{\partial t} = 0$$

pro konzervativní síly $F_i\dot{x}_i = -\frac{\partial U(\vec{x})}{\partial x_i}\dot{x}_i = -\frac{\hat{d} U(\vec{x})}{dt}$

$$m_i\ddot{x}_i\dot{x}_i = F_i\dot{x}_i + \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial x_i}\dot{x}_i + \sum_{l=1}^{p}\mu_l a_{li}\dot{x}_i \Rightarrow \frac{\hat{d} T}{dt} = \vec{F}\cdot\dot{\vec{x}} - \sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial t} - \sum_{l=1}^{p}\mu_l b_l \Rightarrow \frac{\hat{d}}{dt}(T+U) = -\sum_{k=1}^{r}\lambda_k\frac{\partial f_k}{\partial t} - \sum_{l=1}^{p}\mu_l b_l$$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j} \delta q_j$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j} \delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i) \delta x_i = \sum_{i=1}^{3N} \sum_{j=1}^{s} \left[F_i - \frac{\hat{d}}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) \right] \frac{\partial \hat{x}_i}{\partial q_j} \delta q_j = \sum_{i=1}^{3N} \sum_{j=1}^{s} \left[F_i \frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) \frac{\partial \hat{x}_i}{\partial q_j} \right] \delta q_j =$$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i - \widehat{\frac{d}{dt}}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \widehat{\frac{d}{dt}}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

$$= \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \widehat{\frac{d}{dt}}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\widehat{\frac{d}{dt}}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[Q_j - \widehat{\frac{d}{dt}}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{\Delta}\left[Q_j - \widehat{\frac{d}{dt}}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\delta q_j$$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i \ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{\hat{s}}\left[F_i - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\hat{s}}\left[F_i \frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

jsou nezávislé

$$= \sum_{i=1}^{3N}\sum_{j=1}^{\hat{s}}\left[F_i \frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\hat{s}}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{\hat{s}}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\delta q_j$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{T}}{\partial q_j} = Q_j \quad \forall j \in \hat{s}$$

$s = 3N - r$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i \ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

jsou nezávislé

$$= \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{s}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\delta q_j$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{T}}{\partial q_j} = Q_j \quad \forall j \in \hat{s}$$

$s = 3N - r$

$$Q_j = Q_j^{(nep)} + Q_j^{(pot)} = Q_j^{(nep)} + \left[-\frac{\partial U}{\partial x_i} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j} = Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) =$$

$$= Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) = Q_j^{(nep)} - \frac{\partial \hat{U}}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial \hat{U}}{\partial \dot{q}_j}\right)$$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t) \qquad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i \ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

jsou nezávislé

$$= \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{s}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\delta q_j$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{T}}{\partial q_j} = Q_j \quad \forall j \in \hat{s}$$

$s = 3N - r$

$$Q_j = Q_j^{(nep)} + Q_j^{(pot)} = Q_j^{(nep)} + \left[-\frac{\partial U}{\partial x_i} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j} = Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) =$$

$$= Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) = Q_j^{(nep)} - \frac{\partial \hat{U}}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial \hat{U}}{\partial \dot{q}_j}\right)$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{L}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{L}}{\partial q_j} = Q_j^{(nep)}$$

$\hat{L} = \hat{T} - \hat{U}$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

jsou nezávislé

$$= \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{s}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{s}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\delta q_j$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{T}}{\partial q_j} = Q_j \quad \forall j \in \hat{s}$$

$s = 3N - r$

$$Q_j = Q_j^{(nep)} + Q_j^{(pot)} = Q_j^{(nep)} + \left[-\frac{\partial U}{\partial x_i} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j} = Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) =$$

$$= Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) = Q_j^{(nep)} - \frac{\partial \hat{U}}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial \hat{U}}{\partial \dot{q}_j}\right)$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{L}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{L}}{\partial q_j} = Q_j^{(nep)} \qquad \hat{L} = \hat{T} - \hat{U}$$

Pozn: Záměnnost variace a derivace $\frac{\hat{d}}{dt}(\delta\vec{x}) = \frac{\hat{d}}{dt}(\vec{x}' - \vec{x}) = \dot{\vec{x}}' - \dot{\vec{x}} = \delta\dot{\vec{x}} = \delta\left(\frac{\hat{d}\vec{x}}{dt}\right)$ platí i pro funkce $\frac{\hat{d}}{dt}\delta f = \delta\frac{\hat{d}f}{dt}$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\qquad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

jsou nezávislé

$$= \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{\Delta}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\delta q_j$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{T}}{\partial q_j} = Q_j \quad \forall j \in \hat{\Delta}$$

$\Delta = 3N - r$

$$Q_j = Q_j^{(nep)} + Q_j^{(pot)} = Q_j^{(nep)} + \left[-\frac{\partial U}{\partial x_i} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j} = Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) =$$

$$= Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) = Q_j^{(nep)} - \frac{\partial \hat{U}}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial \hat{U}}{\partial \dot{q}_j}\right)$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{L}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{L}}{\partial q_j} = Q_j^{(nep)} \qquad \hat{L} = \hat{T} - \hat{U}$$

Pozn: Záměnnost variace a derivace $\frac{\hat{d}}{dt}(\delta\vec{x}) = \frac{\hat{d}}{dt}(\vec{x}' - \vec{x}) = \dot{\vec{x}}' - \dot{\vec{x}} = \delta\dot{\vec{x}} = \delta\left(\frac{\hat{d}\vec{x}}{dt}\right)$ platí i pro funkce $\frac{\hat{d}}{dt}\delta f = \delta\frac{\hat{d}f}{dt}$

Jourdainův princip

$$0 = \frac{\hat{d}}{dt}\delta A_{ef} = \sum_{i=1}^{3N}\frac{\hat{d}}{dt}(F_i - m_i\ddot{x}_i)\underbrace{\delta x_i}_{\text{zvolíme } = 0} + \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta\dot{x}_i$$

$$\boxed{\sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\cdot\delta_1\dot{x}_i = 0}$$

Jourdainovy variace $\delta_1 t = 0 \quad \delta_1 x_i = 0$

Odvození LR2D pro holonomní soustavu: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$

$$0 = \delta A_{ef} = \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta x_i = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)\frac{\partial \hat{x}_i}{\partial q_j}\right]\delta q_j =$$

$$= \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[F_i\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{i=1}^{3N}\sum_{j=1}^{\Delta}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) + \frac{\partial T}{\partial \dot{x}_i}\left(\frac{\partial \dot{\hat{x}}_i}{\partial q_j}\right)\right]\delta q_j = \sum_{j=1}^{\Delta}\left[Q_j - \frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) + \frac{\partial \hat{T}}{\partial q_j}\right]\overbrace{\delta q_j}^{\text{jsou nezávislé}}$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{T}}{\partial q_j} = Q_j \quad \forall j \in \hat{\Delta}$$

$\Delta = 3N - r$

$$Q_j = Q_j^{(nep)} + Q_j^{(pot)} = Q_j^{(nep)} + \left[-\frac{\partial U}{\partial x_i} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\right)\right]\frac{\partial \hat{x}_i}{\partial q_j} = Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\right) - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) =$$

$$= Q_j^{(nep)} - \frac{\partial U}{\partial x_i}\frac{\partial \hat{x}_i}{\partial q_j} - \frac{\partial U}{\partial \dot{x}_i}\frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}_i}{\partial q_j}\right) + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\right) = Q_j^{(nep)} - \frac{\partial \hat{U}}{\partial q_j} + \frac{\hat{d}}{dt}\left(\frac{\partial \hat{U}}{\partial \dot{q}_j}\right)$$

$$\frac{\hat{d}}{dt}\left(\frac{\partial \hat{L}}{\partial \dot{q}_j}\right) - \frac{\partial \hat{L}}{\partial q_j} = Q_j^{(nep)} \quad \leftarrow \quad \hat{L} = \hat{T} - \hat{U}$$

Pozn: Záměnnost variace a derivace $\frac{\hat{d}}{dt}(\delta\vec{x}) = \frac{\hat{d}}{dt}(\vec{x}' - \vec{x}) = \dot{\vec{x}}' - \dot{\vec{x}} = \delta\dot{\vec{x}} = \delta\left(\frac{\hat{d}\vec{x}}{dt}\right)$ platí i pro funkce $\frac{\hat{d}}{dt}\delta f = \delta\frac{\hat{d}f}{dt}$

Jourdainův princip

$$0 = \frac{\hat{d}}{dt}\delta A_{ef} = \sum_{i=1}^{3N}\frac{\hat{d}}{dt}(F_i - m_i\ddot{x}_i)\underbrace{\delta x_i}_{\text{zvolíme } = 0} + \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta\dot{x}_i$$

$$\boxed{\sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\cdot\delta_1\dot{x}_i = 0}$$

Jourdainovy variace $\delta_1 t = 0 \quad \delta_1 x_i = 0$

Gaussův princip

$$0 = \sum_{i=1}^{3N}\frac{\hat{d}}{dt}(F_i - m_i\ddot{x}_i)\underbrace{\delta_1\dot{x}_i}_{\text{zvolíme } = 0} + \sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\delta_1\ddot{x}_i$$

$$\boxed{\sum_{i=1}^{3N}(F_i - m_i\ddot{x}_i)\cdot\delta_2\ddot{x}_i = 0}$$

Gaussovy variace $\delta_2 t = 0 \quad \delta_2 x_i = 0 \quad \delta_2\dot{x}_i = 0$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i \,\delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i \delta x_i =$$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i \delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i \delta x_i =$$

$$\ddot{x}_i \delta x_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \frac{\hat{d}}{dt}(\delta x_i) = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \delta \dot{x}_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \frac{1}{2}\delta \dot{x}_i^2$$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\,\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i\,\delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i\,\delta x_i = \delta A - \frac{\hat{d}}{dt}\Big(\sum_{i=1}^{3N} \underbrace{m_i \dot{x}_i}_{\frac{\partial T}{\partial \dot{x}_i}}\,\delta x_i\Big) + \delta\Big(\underbrace{\sum_{i=1}^{3N} \frac{1}{2} m_i \dot{x}_i^2}_{T}\Big) = \delta A - \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\,\delta x_i\Big) + \delta T$$

$$\ddot{x}_i\,\delta x_i = \frac{\hat{d}}{dt}(\dot{x}_i\,\delta x_i) - \dot{x}_i \frac{\hat{d}}{dt}(\delta x_i) = \frac{\hat{d}}{dt}(\dot{x}_i\,\delta x_i) - \dot{x}_i\,\delta \dot{x}_i = \frac{\hat{d}}{dt}(\dot{x}_i\,\delta x_i) - \frac{1}{2}\,\delta \dot{x}_i^2$$

$$\boxed{\delta T + \delta A = \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\,\delta x_i\Big)}$$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i \delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i \delta x_i = \delta A - \frac{\hat{d}}{dt}\Big(\sum_{i=1}^{3N} \underbrace{m_i \dot{x}_i}_{\frac{\partial T}{\partial \dot{x}_i}} \delta x_i\Big) + \delta\Big(\underbrace{\sum_{i=1}^{3N} \frac{1}{2} m_i \dot{x}_i^2}_{T}\Big) = \delta A - \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big) + \delta T$$

$$\ddot{x}_i \delta x_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \frac{\hat{d}}{dt}(\delta x_i) = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \delta \dot{x}_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \frac{1}{2}\delta \dot{x}_i^2$$

$$\boxed{\delta T + \delta A = \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big)}$$

v obecných souřadnicích: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$ $\quad \hat{T}(\vec{q}, \dot{\vec{q}}, t) = T(\dot{\hat{\vec{x}}}(\vec{q}, \dot{\vec{q}}, t))$

$$\frac{\partial T}{\partial \dot{x}_i}\delta x_i = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\delta q_j = \frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j \qquad \delta A = F_i \delta x_i = F_i \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = Q_j \delta q_j = \delta \hat{A}$$

$$\boxed{\delta \hat{T} + \delta \hat{A} = \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j\Big)}$$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i \delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i \delta x_i = \delta A - \frac{\hat{d}}{dt}\Big(\sum_{i=1}^{3N} \underbrace{m_i \dot{x}_i}_{\frac{\partial T}{\partial \dot{x}_i}} \delta x_i\Big) + \delta\Big(\underbrace{\sum_{i=1}^{3N} \frac{1}{2} m_i \dot{x}_i^2}_{T}\Big) = \delta A - \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big) + \delta T$$

$$\ddot{x}_i \delta x_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \frac{\hat{d}}{dt}(\delta x_i) = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \delta \dot{x}_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \frac{1}{2}\delta \dot{x}_i^2$$

$$\boxed{\delta T + \delta A = \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big)}$$

v obecných souřadnicích: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$ $\quad \hat{T}(\vec{q}, \dot{\vec{q}}, t) = T(\dot{\hat{\vec{x}}}(\vec{q}, \dot{\vec{q}}, t))$

$$\frac{\partial T}{\partial \dot{x}_i}\delta x_i = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\delta q_j = \frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j \qquad \delta A = F_i \delta x_i = F_i \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = Q_j \delta q_j = \delta \hat{A}$$

$$\boxed{\delta \hat{T} + \delta \hat{A} = \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j\Big)}$$

$$\delta \hat{A} = Q_j^{(nep)}\delta q_j + Q_j^{(pot)}\delta q_j = Q_j^{(nep)}\delta q_j + \Big(\frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\Big) - \frac{\partial \hat{U}}{\partial q_j}\Big)\delta q_j = Q_j^{(nep)}\delta q_j + \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\delta q_j\Big) - \frac{\partial \hat{U}}{\partial \dot{q}_j}\delta \dot{q}_j - \frac{\partial \hat{U}}{\partial q_j}\delta q_j = Q_j^{(nep)}\delta q_j + \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\delta q_j\Big) - \delta \hat{U}$$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i \delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i \delta x_i = \delta A - \frac{\hat{d}}{dt}\Big(\sum_{i=1}^{3N} \underbrace{m_i \dot{x}_i}_{\frac{\partial T}{\partial \dot{x}_i}} \delta x_i\Big) + \delta\Big(\underbrace{\sum_{i=1}^{3N} \frac{1}{2} m_i \dot{x}_i^2}_{T}\Big) = \delta A - \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big) + \delta T$$

$$\ddot{x}_i \delta x_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \frac{\hat{d}}{dt}(\delta x_i) = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \delta \dot{x}_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \frac{1}{2}\delta \dot{x}_i^2$$

$$\boxed{\delta T + \delta A = \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big)}$$

v obecných souřadnicích: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j$ $\quad \hat{T}(\vec{q}, \dot{\vec{q}}, t) = T(\dot{\vec{\hat{x}}}(\vec{q}, \dot{\vec{q}}, t))$

$$\frac{\partial T}{\partial \dot{x}_i}\delta x_i = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\delta q_j = \frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j \qquad \delta A = F_i \delta x_i = F_i \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = Q_j \delta q_j = \delta \hat{A}$$

$$\boxed{\delta \hat{T} + \delta \hat{A} = \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j\Big)}$$

$$\delta \hat{A} = Q_j^{(nep)}\delta q_j + Q_j^{(pot)}\delta q_j = Q_j^{(nep)}\delta q_j + \Big(\frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\Big) - \frac{\partial \hat{U}}{\partial q_j}\Big)\delta q_j = Q_j^{(nep)}\delta q_j + \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\delta q_j\Big) - \frac{\partial \hat{U}}{\partial \dot{q}_j}\delta \dot{q}_j - \frac{\partial \hat{U}}{\partial q_j}\delta q_j = Q_j^{(nep)}\delta q_j + \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\delta q_j\Big) - \delta \hat{U}$$

$$\delta \hat{T} - \delta \hat{U} + Q_j^{(nep)}\delta q_j = \frac{\hat{d}}{dt}\Big(\frac{\partial(\hat{T} - \hat{U})}{\partial \dot{q}_j}\delta q_j\Big) \qquad \boxed{\delta \hat{L} + Q_j^{(nep)}\delta q_j = \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{L}}{\partial \dot{q}_j}\delta q_j\Big)}$$

Ústřední rovnice Lagrangeova – jiný tvar d'Alembertova principu

$$0 = \delta A_{ef} = \sum_{i=1}^{3N} (F_i - m_i \ddot{x}_i)\delta x_i = \underbrace{\sum_{i=1}^{3N} F_i \delta x_i}_{\delta A \text{ virtuální práce akčních sil}} - \sum_{i=1}^{3N} m_i \ddot{x}_i \delta x_i = \delta A - \frac{\hat{d}}{dt}\Big(\sum_{i=1}^{3N} \underbrace{m_i \dot{x}_i}_{\frac{\partial T}{\partial \dot{x}_i}} \delta x_i\Big) + \delta\Big(\underbrace{\sum_{i=1}^{3N} \frac{1}{2} m_i \dot{x}_i^2}_{T}\Big) = \delta A - \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big) + \delta T$$

$$\ddot{x}_i \delta x_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \frac{\hat{d}}{dt}(\delta x_i) = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \dot{x}_i \delta \dot{x}_i = \frac{\hat{d}}{dt}(\dot{x}_i \delta x_i) - \frac{1}{2}\delta \dot{x}_i^2$$

$$\boxed{\delta T + \delta A = \frac{\hat{d}}{dt}\Big(\frac{\partial T}{\partial \dot{x}_i}\delta x_i\Big)}$$

v obecných souřadnicích: $x_i = \hat{x}_i(\vec{q}, t)$ $\quad \delta x_i = \delta \hat{x}_i = \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j \quad \hat{T}(\vec{q}, \dot{\vec{q}}, t) = T(\dot{\vec{\hat{x}}}(\vec{q}, \dot{\vec{q}}, t))$

$$\frac{\partial T}{\partial \dot{x}_i}\delta x_i = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = \frac{\partial T}{\partial \dot{x}_i}\frac{\partial \dot{\hat{x}}_i}{\partial \dot{q}_j}\delta q_j = \frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j \qquad \delta A = F_i \delta x_i = F_i \frac{\partial \hat{x}_i}{\partial q_j}\delta q_j = Q_j \delta q_j = \delta \hat{A}$$

$$\boxed{\delta \hat{T} + \delta \hat{A} = \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{T}}{\partial \dot{q}_j}\delta q_j\Big)}$$

$$\delta \hat{A} = Q_j^{(nep)}\delta q_j + Q_j^{(pot)}\delta q_j = Q_j^{(nep)}\delta q_j + \Big(\frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\Big) - \frac{\partial \hat{U}}{\partial q_j}\Big)\delta q_j = Q_j^{(nep)}\delta q_j + \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\delta q_j\Big) - \frac{\partial \hat{U}}{\partial \dot{q}_j}\delta \dot{q}_j - \frac{\partial \hat{U}}{\partial q_j}\delta q_j = Q_j^{(nep)}\delta q_j + \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{U}}{\partial \dot{q}_j}\delta q_j\Big) - \delta \hat{U}$$

$$\delta \hat{T} - \delta \hat{U} + Q_j^{(nep)}\delta q_j = \frac{\hat{d}}{dt}\Big(\frac{\partial(\hat{T} - \hat{U})}{\partial \dot{q}_j}\delta q_j\Big) \qquad \boxed{\delta \hat{L} + Q_j^{(nep)}\delta q_j = \frac{\hat{d}}{dt}\Big(\frac{\partial \hat{L}}{\partial \dot{q}_j}\delta q_j\Big)}$$

Pozn: diferenciální počet

Funkce přírůstek funkce lineární část přírůstku = diferenciál funkce derivace stacionární hodnota

$f: \mathbb{R} \to \mathbb{R}$

$$\Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\,\Delta x}^{} + \underbrace{\omega(x,\Delta x)}_{\to 0 \text{ pro } \Delta x \to 0} \cdot \Delta x$$

$$df = A(x)\Delta x = f'(x)dx$$

$f(x) = x \quad dx = \Delta x$

$$f'(x) = \frac{df}{dx} = A(x)$$

$$f'(x_0) = 0$$

Funkce	přírůstek funkce	lineární část přírůstku = diferenciál funkce	derivace	stacionární hodnota
$f:\mathbb{R}\to\mathbb{R}$	$\Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\Delta x} + \underbrace{\omega(x,\Delta x)}_{\downarrow\, 0 \text{ pro } \Delta x\to 0}\cdot\Delta x$	$df = A(x)\Delta x = f'(x)dx$ $f(x)=x \quad dx=\Delta x$	$f'(x)=\frac{df}{dx}=A(x)$	$f'(x_0)=0$
$f:\mathbb{R}^m\to\mathbb{R}$	$\Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\downarrow\, 0 \text{ pro } \lvert\Delta\vec{x}\rvert\to 0}\cdot\lvert\Delta\vec{x}\rvert$	$df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$	$f'(\vec{x}) = A(\vec{x})\in\mathcal{L}(\mathbb{R}^m,\mathbb{R})$ $A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$	$\frac{\partial f(\vec{x}_0)}{\partial x_j} = 0$ $\forall j\in\hat{m}$

Funkce	přírůstek funkce	lineární část přírůstku = diferenciál funkce	derivace	stacionární hodnota
$f:\mathbb{R}\to\mathbb{R}$	$\Delta f = f(x+\Delta x) - f(x) = A(x)\Delta x + \underbrace{\omega(x,\Delta x)}_{\downarrow\, 0 \text{ pro } \Delta x\to 0}\cdot\Delta x$	$df = A(x)\Delta x = f'(x)dx$ $f(x)=x \quad dx=\Delta x$	$f'(x)=\frac{df}{dx}=A(x)$	$f'(x_0)=0$
$f:\mathbb{R}^n\to\mathbb{R}$	$\Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{n} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\downarrow\, 0 \text{ pro } \lvert\Delta\vec{x}\rvert\to 0}\cdot\lvert\Delta\vec{x}\rvert$	$df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{n}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$ pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big\vert_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$	$f'(\vec{x}) = A(\vec{x})\in\mathcal{L}(\mathbb{R}^n,\mathbb{R})$ $A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$	$\frac{\partial f(\vec{x}_0)}{\partial x_j}=0$ $\forall j\in\hat{n}$

Funkce | přírůstek funkce | lineární část přírůstku = diferenciál funkce | derivace | stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\quad \Delta f = f(x+\Delta x) - f(x) = A(x)\Delta x + \underbrace{\omega(x,\Delta x)}_{\downarrow 0 \text{ pro } \Delta x\to 0}\cdot\Delta x$

$df = A(x)\Delta x = f'(x)dx$

$f(x)=x \qquad dx = \Delta x$

$f'(x) = \frac{df}{dx} = A(x)$

$f'(x_0) = 0$

$f:\mathbb{R}^m\to\mathbb{R}$ $\quad \Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\downarrow 0 \text{ pro } |\Delta\vec{x}|\to 0}\cdot|\Delta\vec{x}|$

$df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$

$f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^m,\mathbb{R})$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$

$\frac{\partial f(\vec{x}_0)}{\partial x_j} = 0 \quad \forall j\in\hat{m}$

pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet $\quad$ body $\to$ křivky $\quad$ funkce $\to$ funkcionály

Křivka (třídy $r\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

Funkce přírůstek funkce lineární část přírůstku = diferenciál funkce derivace stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\Delta x}^{} + \underbrace{\omega(x,\Delta x)}_{\downarrow \text{ pro } \Delta x\to 0 \atop 0}\cdot\Delta x$ $df = A(x)\Delta x = f'(x)dx$ $f'(x) = \frac{df}{dx} = A(x)$ $f'(x_0) = 0$

$f(x) = x \quad dx = \Delta x$

$f:\mathbb{R}^n\to\mathbb{R}$ $\Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{n} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\downarrow \text{ pro } |\Delta\vec{x}|\to 0 \atop 0}\cdot|\Delta\vec{x}|$ $df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{n}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$ $f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^n,\mathbb{R})$ $\frac{\partial f(\vec{x}_0)}{\partial x_j} = 0$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$ $\forall j\in\hat{n}$

pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet body $\to$ křivky funkce $\to$ funkcionály

Křivka (třídy $n\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(n)}$ tj. má spojité derivace do řádu n).

$C^{(n)}\langle a,b\rangle$ mn. všech křivek třídy n tvoří vektorový prostor dim $+\infty$

s normu $\|\varphi\| = \max_{x\in\langle a,b\rangle}\{|\varphi(x)|,|\varphi'(x)|,\dots,|\varphi^{(n)}(x)|\}$ který označíme $\vec{C}^{(n)}\langle a,b\rangle$

Funkce   přírůstek funkce   lineární část přírůstku = diferenciál funkce   derivace   stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\Delta f = f(x+\Delta x) - f(x) = A(x)\Delta x + \underbrace{\omega(x,\Delta x)}_{\substack{\downarrow \text{ pro } \Delta x\to 0\\ 0}}\cdot\Delta x$ $df = A(x)\Delta x = f'(x)dx$ $f'(x) = \frac{df}{dx} = A(x)$ $f'(x_0) = 0$

$f(x)=x \quad dx = \Delta x$

$f:\mathbb{R}^m\to\mathbb{R}$ $\Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\substack{\downarrow \text{ pro } |\Delta\vec{x}|\to 0\\ 0}}\cdot|\Delta\vec{x}|$ $df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$

$f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^m,\mathbb{R})$ $\frac{\partial f(\vec{x}_0)}{\partial x_j} = 0$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$ $\forall j\in\hat{m}$

pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet   body $\to$ křivky   funkce $\to$ funkcionály

Křivka (třídy $r\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

$C^{(r)}\langle a,b\rangle$ mn. všech křivek třídy r tvoří vektorový prostor dim $+\infty$

s normu $\|\varphi\| = \max\limits_{x\in\langle a,b\rangle}\{|\varphi(x)|,|\varphi'(x)|,\dots,|\varphi^{(r)}(x)|\}$ který označíme $\vec{C}^{(r)}\langle a,b\rangle$

$C^{(r)}_{(A,B)}\langle a,b\rangle = \{\varphi\in C^{(r)}\langle a,b\rangle \,|\, \varphi(a)=A \wedge \varphi(b)=B\}$ mn. křivek z A do B

$\left(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle\right)$ normovaný afinní prostor

Funkce | přírůstek funkce | lineární část přírůstku = diferenciál funkce | derivace | stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\quad \Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\Delta x} + \underbrace{\omega(x,\Delta x)}_{\downarrow\ \text{pro}\ \Delta x\to 0 \atop 0}\cdot\Delta x$ $\quad df = A(x)\Delta x = f'(x)dx$ $\quad f'(x)=\frac{df}{dx}=A(x)$ $\quad f'(x_0)=0$

$f(x)=x \quad dx=\Delta x$

$f:\mathbb{R}^m\to\mathbb{R}$ $\quad \Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\downarrow\ \text{pro}\ |\Delta\vec{x}|\to 0 \atop 0}\cdot|\Delta\vec{x}|$ $\quad df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$ $\quad f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^m,\mathbb{R})$ $\quad \frac{\partial f(\vec{x}_0)}{\partial x_j}=0$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j \qquad \forall j\in\hat{m}$

pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet $\quad$ body $\to$ křivky $\quad$ funkce $\to$ funkcionály

Křivka (třídy $r\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

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$C^{(r)}_{(A,B)}\langle a,b\rangle = \{\varphi\in C^{(r)}\langle a,b\rangle \mid \varphi(a)=A \wedge \varphi(b)=B\}$ mn. křivek z A do B

$(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle)$ normovaný afinní prostor

Buď $\varphi\in C^{(r)}_{(A,B)}\langle a,b\rangle$ pak pro lib. $\overline{\varphi}\in C^{(r)}_{(A,B)}\langle a,b\rangle$ nazýváme $\delta\varphi = \overline{\varphi}-\varphi \in \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle$ variací křivky φ s pevnými konci.

$C^{(r)}\langle a,b\rangle \qquad \vec{C}^{(r)}\langle a,b\rangle$ s volnými konci.

Funkce | přírůstek funkce | lineární část přírůstku = diferenciál funkce | derivace | stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\quad \Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\Delta x} + \underbrace{\omega(x,\Delta x)}_{\downarrow\, 0 \text{ pro } \Delta x\to 0}\cdot\Delta x$ $\quad df = A(x)\Delta x = f'(x)dx$ $\quad f'(x) = \frac{df}{dx} = A(x)$ $\quad f'(x_0) = 0$

$f(x)=x \quad dx = \Delta x$

$f:\mathbb{R}^m\to\mathbb{R}$ $\quad \Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\downarrow\, 0 \text{ pro } |\Delta\vec{x}|\to 0}\cdot|\Delta\vec{x}|$ $\quad df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$

$f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^m,\mathbb{R})$ $\quad A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$ $\quad \frac{\partial f(\vec{x}_0)}{\partial x_j} = 0 \quad \forall j\in\hat{m}$

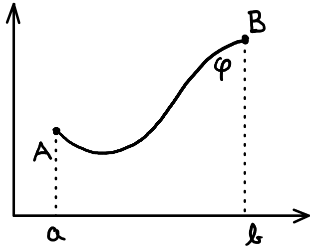
pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet $\quad$ body → křivky $\quad$ funkce → funkcionály

Křivka (třídy $r\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

$C^{(r)}\langle a,b\rangle$ mn. všech křivek třídy r tvoří vektorový prostor dim $+\infty$

s normu $\|\varphi\| = \max_{x\in\langle a,b\rangle}\{|\varphi(x)|, |\varphi'(x)|, \dots, |\varphi^{(r)}(x)|\}$ který označíme $\vec{C}^{(r)}\langle a,b\rangle$



$C^{(r)}_{(A,B)}\langle a,b\rangle = \{\varphi\in C^{(r)}\langle a,b\rangle \mid \varphi(a)=A \wedge \varphi(b)=B\}$ mn. křivek z A do B

$(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle)$ normovaný afinní prostor

Buď $\varphi\in C^{(r)}_{(A,B)}\langle a,b\rangle$ pak pro lib. $\bar{\varphi}\in C^{(r)}_{(A,B)}\langle a,b\rangle$ nazýváme $\delta\varphi = \bar{\varphi}-\varphi \in \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle$ variací křivky φ s pevnými konci.

$C^{(r)}\langle a,b\rangle \qquad \vec{C}^{(r)}\langle a,b\rangle$ s volnými konci.

Funkce | přírůstek funkce | lineární část přírůstku = diferenciál funkce | derivace | stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\quad \Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\,\Delta x}^{} + \underbrace{\omega(x,\Delta x)}_{\to 0 \text{ pro } \Delta x \to 0}\cdot \Delta x$ $\quad df = A(x)\Delta x = f'(x)dx$ $\quad f'(x) = \frac{df}{dx} = A(x)$ $\quad f'(x_0) = 0$

$f(x) = x \quad dx = \Delta x$

$f:\mathbb{R}^n\to\mathbb{R}$ $\quad \Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{n} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\to 0 \text{ pro } |\Delta\vec{x}| \to 0}\cdot|\Delta\vec{x}|$ $\quad df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{n} \frac{\partial f(\vec{x})}{\partial x_j} dx_j$

$f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^n, \mathbb{R})$ $\quad \frac{\partial f(\vec{x}_0)}{\partial x_j} = 0$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$ $\quad \forall j \in \hat{n}$

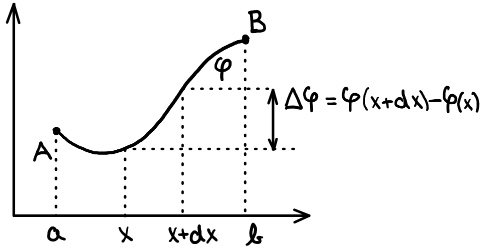
pokud existuje, pak $df = \frac{\partial}{\partial \varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon\, d\vec{x})$

Variační počet $\quad$ body → křivky $\quad$ funkce → funkcionály

Křivka (třídy $r \in \mathbb{N}_0$) je spojité zobrazení $\varphi: \langle a,b\rangle \subset \mathbb{R} \to \mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

$C^{(r)}\langle a,b\rangle$ mn. všech křivek třídy r tvoří vektorový prostor dim $+\infty$

s normu $\|\varphi\| = \max_{x\in\langle a,b\rangle}\{|\varphi(x)|, |\varphi'(x)|, \dots, |\varphi^{(r)}(x)|\}$ který označíme $\vec{C}^{(r)}\langle a,b\rangle$



$C^{(r)}_{(A,B)}\langle a,b\rangle = \{\varphi \in C^{(r)}\langle a,b\rangle \mid \varphi(a) = A \wedge \varphi(b) = B\}$ mn. křivek z A do B

$\left(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle\right)$ normovaný afinní prostor

Buď $\varphi \in C^{(r)}_{(A,B)}\langle a,b\rangle$ pak pro lib. $\bar{\varphi} \in C^{(r)}_{(A,B)}\langle a,b\rangle$ nazýváme $\delta\varphi = \bar{\varphi} - \varphi \in \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle$ variací křivky φ s pevnými konci.

$C^{(r)}\langle a,b\rangle \qquad \vec{C}^{(r)}\langle a,b\rangle \qquad$ s volnými konci.

Funkce   přírůstek funkce   lineární část přírůstku = diferenciál funkce   derivace   stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\Delta x} + \underbrace{\omega(x,\Delta x)}_{\substack{\downarrow \text{ pro } \Delta x\to 0\\ 0}}\cdot\Delta x$ $df = A(x)\Delta x = f'(x)dx$ $f'(x) = \frac{df}{dx} = A(x)$ $f'(x_0) = 0$

$f(x) = x \quad dx = \Delta x$

$f:\mathbb{R}^m\to\mathbb{R}$ $\Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\substack{\downarrow \text{ pro } |\Delta\vec{x}|\to 0\\ 0}}\cdot|\Delta\vec{x}|$ $df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$

$f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^m,\mathbb{R})$ $\frac{\partial f(\vec{x}_0)}{\partial x_j} = 0$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$ $\forall j\in\hat{m}$

pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet   body $\to$ křivky   funkce $\to$ funkcionály

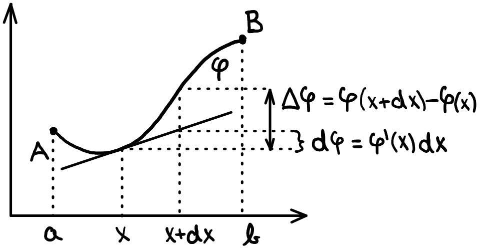
Křivka (třídy $r\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

$C^{(r)}\langle a,b\rangle$ mn. všech křivek třídy r tvoří vektorový prostor dim $+\infty$

s normu $\|\varphi\| = \max_{x\in\langle a,b\rangle}\{|\varphi(x)|,|\varphi'(x)|,\dots,|\varphi^{(r)}(x)|\}$ který označíme $\vec{C}^{(r)}\langle a,b\rangle$

$C^{(r)}_{(A,B)}\langle a,b\rangle = \{\varphi\in C^{(r)}\langle a,b\rangle \,|\, \varphi(a)=A \wedge \varphi(b)=B\}$ mn. křivek z A do B

$(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle)$ normovaný afinní prostor



Buď $\varphi\in C^{(r)}_{(A,B)}\langle a,b\rangle$ pak pro lib. $\overline{\varphi}\in C^{(r)}_{(A,B)}\langle a,b\rangle$ nazýváme $\delta\varphi = \overline{\varphi} - \varphi \in \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle$ variací křivky φ s pevnými konci.

$C^{(r)}\langle a,b\rangle$ $\vec{C}^{(r)}\langle a,b\rangle$ s volnými konci.

Funkce přírůstek funkce lineární část přírůstku = diferenciál funkce derivace stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\Delta f = f(x+\Delta x) - f(x) = A(x)\Delta x + \underbrace{\omega(x,\Delta x)}_{\to 0 \text{ pro } \Delta x \to 0}\cdot\Delta x$ $df = A(x)\Delta x = f'(x)dx$ $f'(x) = \frac{df}{dx} = A(x)$ $f'(x_0) = 0$

$f(x) = x$ $dx = \Delta x$

$f:\mathbb{R}^m\to\mathbb{R}$ $\Delta f = f(\vec{x}+\Delta\vec{x}) - f(\vec{x}) = \sum_{j=1}^{m} A_j(\vec{x})\Delta x_j + \underbrace{\omega(\vec{x},\Delta\vec{x})}_{\to 0 \text{ pro } |\Delta\vec{x}|\to 0}\cdot|\Delta\vec{x}|$ $df = \underbrace{A(\vec{x})\Delta\vec{x}}_{\nabla f(\vec{x})\cdot d\vec{x}} = \sum_{j=1}^{m}\frac{\partial f(\vec{x})}{\partial x_j}dx_j$ $f'(\vec{x}) = A(\vec{x}) \in \mathcal{L}(\mathbb{R}^m,\mathbb{R})$ $\frac{\partial f(\vec{x}_0)}{\partial x_j} = 0$

$A_j(\vec{x}) = \frac{\partial f}{\partial x_j} = \partial_j f = f_j$ $\forall j \in \hat{m}$

pokud existuje, pak $df = \frac{\partial}{\partial\varepsilon}\Big|_{\varepsilon=0} f(\vec{x}+\varepsilon d\vec{x})$

Variační počet body → křivky funkce → funkcionály

Křivka (třídy $r\in\mathbb{N}_0$) je spojité zobrazení $\varphi:\langle a,b\rangle\subset\mathbb{R}\to\mathbb{R}$ (třídy $C^{(r)}$ tj. má spojité derivace do řádu r).

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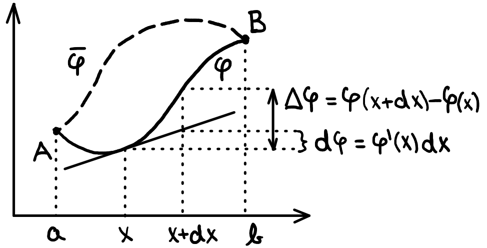
$\left(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle\right)$ normovaný afinní prostor



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Funkce | přírůstek funkce | lineární část přírůstku = diferenciál funkce | derivace | stacionární hodnota

$f:\mathbb{R}\to\mathbb{R}$ $\quad \Delta f = f(x+\Delta x) - f(x) = \overbrace{A(x)\Delta x} + \underbrace{\omega(x,\Delta x)}_{\to 0 \text{ pro } \Delta x\to 0}\cdot\Delta x$ $\quad df = A(x)\Delta x = f'(x)dx$ $\quad f'(x) = \frac{df}{dx} = A(x)$ $\quad f'(x_0) = 0$

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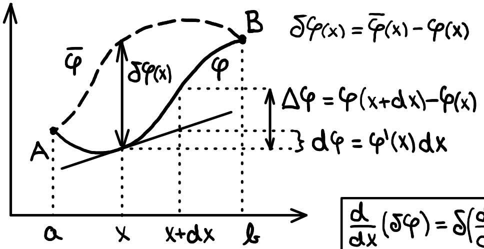

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$\left(C^{(r)}_{(A,B)}\langle a,b\rangle, -, \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle\right)$ normovaný afinní prostor

$\boxed{\frac{d}{dx}(\delta\varphi) = \delta\left(\frac{d\varphi}{dx}\right)}$

Buď $\varphi\in C^{(r)}_{(A,B)}\langle a,b\rangle$ pak pro lib. $\bar{\varphi}\in C^{(r)}_{(A,B)}\langle a,b\rangle$ nazýváme $\delta\varphi = \bar{\varphi} - \varphi \in \vec{C}^{(r)}_{(0,0)}\langle a,b\rangle$ variací křivky φ s pevnými konci.

$C^{(r)}\langle a,b\rangle \qquad \vec{C}^{(r)}\langle a,b\rangle$ s volnými konci.

Funkcionál je zobrazení z prostoru (např. vektorového) do reálných čísel.

Funkcionál $I : C^{(n)}\langle a,b\rangle \to \mathbb{R}$ je spojitý na křivce φ, pokud $\forall \varepsilon > 0 \ \exists \delta > 0 \ \forall \bar{\varphi} \in C^{(n)}\langle a,b\rangle \ \ \|\bar{\varphi}-\varphi\| < \delta \Rightarrow |I(\bar{\varphi}) - I(\varphi)| < \varepsilon$

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tj. $$\Delta I = I(\varphi+\delta\varphi) - I(\varphi) = \underbrace{\Phi[\delta\varphi]}_{\text{lineární část přírustku } \delta I(\varphi)[\delta\varphi]} + \underbrace{\omega(\varphi,\delta\varphi)}_{\to 0 \text{ pro } \|\delta\varphi\| \to 0} \cdot \|\delta\varphi\|$$

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Pozn: existuje-li variace, lze ji najít takto $\eta \in \vec{C}^{(n)}\langle a,b\rangle \quad \hat{I}(\varepsilon) = I(\varphi+\overbrace{\varepsilon\eta}^{\delta\varphi})\quad \forall \varepsilon\in\mathbb{R}$

$$\left.\frac{d\hat{I}}{d\varepsilon}\right|_{\varepsilon=0} = \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\big(I(\varphi+\varepsilon\eta)-I(\varphi)\big) = \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\big(\Phi[\varepsilon\eta]+\omega(\varphi,\varepsilon\eta)\cdot\|\varepsilon\eta\|\big) = \lim_{\varepsilon\to 0}\big(\Phi[\eta]+\operatorname{sgn}(\varepsilon)\,\omega(\varphi,\varepsilon\eta)\cdot\|\eta\|\big) = \Phi(\eta) = \delta I(\varphi)[\eta]$$

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Funkcionál $I : C^{(n)}\langle a,b\rangle \to \mathbb{R}$ má na křivce φ – maximum (minimum) pokud $\forall \bar{\varphi} \in C^{(n)}\langle a,b\rangle\ \ I(\varphi) \geq I(\bar{\varphi})\ \ (I(\varphi) \leq I(\bar{\varphi}))$

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– stacionární hodnotu (φ je extremálou I) pokud $\delta I(\varphi) = 0$