TRANSVERSE MOMENTUM ANALYSIS OF COLLECTIVE MOTION IN RELATIVISTIC NUCLEAR COLLISIONS ☆

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Received 15 March 1985

A novel transverse-momentum technique is used to analyse charged-particle exclusive data for collective motion in the Ar + KCl reaction at 1.8 GeV/nucleon. Previous analysis of this reaction, employing the standard sphericity tensor, revealed no significant effect. In the present analysis, collective effects are observed, and they are substantially stronger than in the Cugnon cascade model, but weaker than in the hydrodynamical model.

Much experimental and theoretical attention in the study of high-energy heavy-ion collisions has been devoted to the possible existence of collective motion, following decompression of highly excited nuclear matter. Evidence has been claimed from two-particle correlations [1] and from sphericity analysis [2]. The extraction of collective flow parameters from data and the comparison with theoretical predictions, such as those of the cascade [3] and hydrodynamic [4] models is made difficult by the existence of statistical fluctuations. To recognize this, it is easy to see that a spherical momentum distribution, populated at random by a finite number of particles, will yield a reaction plane, an elongated sphericity tensor, and a finite collective flow angle. The magnitude of the uninteresting statistical effects will depend on the multiplicity of particles.

In this letter we isolate collective motion in the Ar + KCl (1.8 GeV/nucleon) reaction [5], with a novel transverse-momentum analysis method. Transverse momentum (see also ref. [6]) is selected to avoid any possible effects due to nuclear transparency and corona [7] effects that would be manifested primarily in the longitudinal momenta. We determine the reaction plane in the collisions, and show how to estimate its uncertainty. We show how to remove finite-multiplicity distortions from events rotated to the reaction plane, and present the collective effects as the distribution of average transverse momenta projected onto the reaction plane, as a function of rapidity. By this means we are able to demonstrate collective motion in a reaction for which the sphericity model was inconclusive [5].

The semi-exclusive data of the near-symmetric Ar + KCl (1.8 GeV/nucleon) reaction come from central-trigger measurements in the streamer chamber at the Bevalac. Details of the experiment have been reported previously [5]. The analysed 495 collision events were processed for protons, deuterons, tritons, π^+, and π^- from the reaction. The central-trigger cross section of 180 mb corresponds in the geometric picture to a cutoff in the impact parameter at $b < 2.4$ fm.

We define a vector constructed from the transverse momenta $\boldsymbol{p}_\nu^\perp$ of detected particles:

$$\boldsymbol{Q} = \sum_{\nu=1}^{M} \omega_\nu \boldsymbol{p}_\nu^\perp , \qquad (1)$$

where ν is a particle index and ω_ν is a weight. We choose $\omega_\nu = 0$ for pions. For the baryons we choose

☆ Work supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the US Department of Energy under Contract DE-AC03-76SF00098.

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$\omega_\nu = 1$ for $y_\nu > y_c + \delta$, $\omega_\nu = -1$ for $y_\nu < y_c - \delta$, and $\omega_\nu = 0$ otherwise. For symmetric collisions it is natural to choose y_c equal to the value for the overall CM system, $y_c = y_B/2 = 0.87$. The quantity δ is inserted to remove particles from midrapidity which do not contribute to the determination of the reaction plane but do contribute unwanted fluctuations. For this reaction we take $\delta = 0.3$ which excludes 35% of detected nuclear fragments. The direction of the vector Q can finally be used to estimate the reaction plane in an event, and its magnitude to determine average transverse momentum transfer in the reaction.

To estimate the accuracy of the procedure we divided randomly each event into two, and compared the reaction planes extracted from the two sub-events. The azimuthal correlation between the constructed vectors Q_{I} and Q_{II} is shown in fig. 1a. The fact that the distribution is not flat testifies to the determination of the reaction plane. To verify that this result is not due to inefficiencies in the streamer chamber acceptance, we performed a similar test using Monte Carlo events generated by mixing particles from events

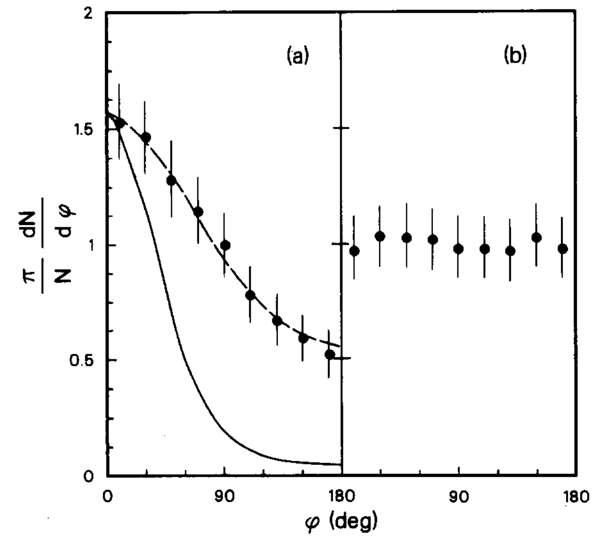

Fig. 1. Azimuthal angle distribution of vectors Q_{I} and Q_{II} from subevents with respect to each other: (a) for data, (b) for Monte Carlo events. The dashed line is for the simulation with two gaussian sources; the solid line represents azimuthal angle distribution of vector Q from the sources with respect to the reaction plane (this is normalized to the height with the sampling distribution).

generated by mixing particles from events within the same multiplicity range. The resulting distribution is shown in fig. 1b. It is completely flat. Arguing with the central-limit theorem and small-angle expansion one can deduce that the distribution of the observed Q with respect to the reaction plane should be more narrow than that of fig. 1a by a factor of 2. (A $\sqrt{2}$ factor in the width reduction comes from the increase in multiplicity, and a $\sqrt{2}$ from the change from a deviation between two sampling vectors, to a deviation from plane.) The optimal choice of δ in the definition of Q can be made to minimize the width of the distribution in fig. 1a. To verify that the result is not dominated by a few particles, we removed from each event the four particles with the highest transverse momenta in Q. The correlation was slightly diminished, but remained within the error bars of fig. 1a.

We turn to a discussion of the magnitude of Q. If Q were just a sum of randomly oriented momenta, then we should have the average $\overline{Q^2} = \overline{\Sigma p^{\perp 2}}$. However, from the data we get

$$\overline{Q^2 - \sum p^{\perp 2}} = \sum_{\mu \neq \nu} \overline{(\omega_\mu p_\mu^\perp)(\omega_\nu p_\nu^\perp)} = 11.4 - 6.7$$
$$= 4.7 \pm 0.5\,(\mathrm{GeV}/c)^2 \ . \qquad (2)$$

To the extent that correlations, other than those stemming from existence of the reaction plane (initial state of the collision), are weak,

$$\overline{Q^2 - \sum p^{\perp 2}} = \sum_{\mu \neq \nu} \overline{(\omega_\mu p_\mu^{\mathrm{x}})}\,\overline{(\omega_\nu p_\nu^{\mathrm{x}})} \ , \qquad (3)$$

with the averages at the rhs of (3) taken in the coordinate system associated with the reaction plane, and x denoting a vector component in the reaction plane. The equation follows an assumption of uncorrelated emission in the coordinate system associated with the reaction plane, viewed as a correlated emission in the system associated with apparatus. With a fragment mass a_ν, and the total mass of fragments with $\omega_\nu \neq 0$, $A = \Sigma a$, we estimate average momenta per nucleon in the reaction plane from

$$\overline{\omega p^{\mathrm{x}}/a} \simeq \left[\left(\overline{Q^2 - \sum p^{\perp 2}}\right)\Big/\left(\overline{A^2 - \sum a^2}\right)\right]^{1/2}$$
$$= 95 \pm 5\ \mathrm{MeV}/c \ , \qquad (4)$$

and the average transverse-momentum transfer in the reaction plane, $\overline{Q^x}$, with

$$\overline{Q^x} = \bar{A} \cdot \overline{\omega p^x/a} = 2.17 \pm 0.11 \text{ GeV}/c \quad (5)$$

[for practical purposes this is the square root of the numerical value of eq. (2)]. Finally, for total transverse momentum in the forward region, $y > y_c + \delta$, in terms of nuclear charges, $\overline{P^x_f}$, we estimate

$$\overline{P^x_f} \simeq \overline{Z_f} \cdot \overline{\omega p^x/a} = 1.04 \pm 0.55 \text{ GeV}/c\,. \quad (6)$$

Utilizing (4), the experimental $\overline{p^{\perp 2}}$, and multiplicities, we attempted to simulate the data with two gaussian sources associated with the reaction plane in the transverse-momentum space. The simulation reproduces nontrivial $p^\perp$ data averages, and also distributions, like dN/dQ^2. Carrying with the sources the procedure as with data for fig. 1a, we get the dashed line in fig. 1a. The solid line in fig. 1a indicates the azimuthal angle distribution of $\boldsymbol{Q}$ from the sources with respect to the reaction plane.

The distribution is broad, $(\overline{\varphi^2})^{1/2} \simeq 56°$. For the subsequent analysis it is important whether $\overline{\cos\varphi}$ is significantly larger than zero as compared with unity, and we get $\overline{\cos\varphi} \simeq 0.65$.

We now proceed to establish the average transverse momentum per nucleon in the reaction plane as a function of rapidity $\overline{p^x/a}(y)$. We start with what might seem most natural [8], rotating events to a common reaction plane, determined for each event by $\boldsymbol{Q}$, and evaluating the in-plane averages $\overline{p^{x'}/a}(y)$. The results are shown in fig. 2a. Since momenta are not projected on the true reaction plane, but on an estimated one, we put a prime on x. In fig. 2b we show results from the same procedure with the Monte Carlo events that lack a dynamic effect in the reaction plane. The

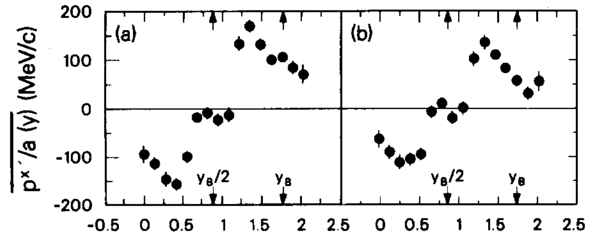

Fig. 2. Average in-plane transverse momentum per nucleon as a function of rapidity, from rotation of events to the reaction plane determined by $\boldsymbol{Q}$: (a) for data, (b) for Monte Carlo events.

figures look similar because of the finite-multiplicity fluctuations [8]. Let us examine the distortion of momenta in fig. 2, exhibited in the apparent collective effect for Monte Carlo events. As we project momenta on the reaction plane from $\boldsymbol{Q}$, we evaluate

$$p^{x'}_\nu = \boldsymbol{p}^\perp_\nu \cdot \boldsymbol{Q}/Q = \boldsymbol{p}^\perp_\nu \cdot \sum_\mu \omega_\mu \boldsymbol{p}^\perp_\mu \Big/ \Big|\sum_\mu \omega_\mu \boldsymbol{p}^\perp_\mu\Big|$$

$$= \Big(\omega_\nu p^{\perp 2}_\nu + \boldsymbol{p}^\perp_\nu \cdot \sum_{\mu\neq\nu} \omega_\mu \boldsymbol{p}^\perp_\mu\Big) \Big/ \Big|\sum_\mu \omega_\mu \boldsymbol{p}^\perp_\mu\Big|\,, \quad (7)$$

and for Monte Carlo events

$$\overline{p^{x'}}(y) \sim \overline{\omega p^{\perp 2}}/\sqrt{M}\overline{p^\perp} = \overline{\omega p^\perp}/\sqrt{M}$$

$$\sim (530 \text{ MeV}/c)/4.5 \sim 100 \text{ MeV}/c\,. \quad (8)$$

Here M stands for the number of particles contributing to $\boldsymbol{Q}$, and we insert into (8) values appropriate for the reaction. The distortion $(\sim 1/\sqrt{M})$ occurs because we project a particle momentum on itself. This is more general: by relating a particle to a construct in which it has been used we always probe a correlation of a particle with itself. The distortion of momenta would, in particular, occur if the reaction plane were estimated from sphericity matrix [2,8].

To remove the distortion of momenta, we determine the reaction plane for each particle separately from the remaining particles in an event,

$$p^{x'}_\nu = \boldsymbol{p}^\perp_\nu \cdot \boldsymbol{Q}_\nu/Q_\nu\,, \quad \boldsymbol{Q}_\nu = \sum_{\mu\neq\nu} \omega_\mu \boldsymbol{p}^\perp_\mu\,. \quad (9)$$

The reevaluated $\overline{p^{x'}/a}(y)$ are shown in figs. 3a, 3b, for data and Monte Carlo events, respectively, and the distinction is now clear. Figs. 3c, 3d, show the average differential, per unit rapidity, transverse momentum deposition in the estimated reaction plane, $\overline{dP^{x'}/dy}$, in terms of nuclear charges. The variable, integrated over a rapidity interval, measures total transverse momentum deposited in nuclear charges in an interval. As the particle momenta are not projected onto the true reaction plane, the average momenta get reduced: $\overline{p^{x'}}(y) = \overline{p^x}(y)\,\overline{\cos\varphi}$, where φ is the azimuthal angle deviation of the estimated plane from the true one. Normalizing momenta with the total observed momentum (5), we find $\overline{Q^{x'}}/\overline{Q^x} = \overline{\cos\varphi} = 0.64 \pm 0.06$, in agreement with the simulated distribution of $\boldsymbol{Q}$ in fig. 1a.

With the result (6) and fig. 3c, we estimate total

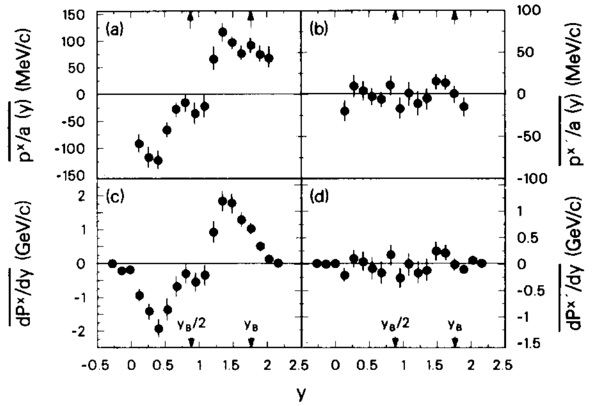

Fig. 3. (a), (b) Average momentum per nucleon in the estimated reaction plane $\overline{p^{x'}/a}(y)$, upon removal of finite-multiplicity distortions, for data and Monte Carlo events, respectively. (c), (d) Differential, per unit rapidity, transverse momentum deposition in the estimated reaction plane in terms of nuclear charges $\overline{dP^{x'}/dy}$, for data and Monte Carlo events, respectively. Left-hand scales in (a) and (c) yield respective estimated average momenta per nucleon $\overline{p^x/a}(y)$, and deposition $\overline{dP^x/dy}$, in the true reaction plane.

transverse momentum transfer between hemispheres ($y \lessgtr y_B/2$) in terms of charges of p, d, t from the reaction, at 2.3 ± 0.2 GeV/c. Assuming a corresponding amount of transfer in neutrons would bring the total transfer in the reaction to $\gtrsim$ 4.9 GeV/c. An estimate, from eq. (13.1) of ref. [9], shows that Coulomb repulsion cannot contribute more than ~10 MeV/c per particle to the reaction-plane transverse momenta of forward and backward going nuclear fragments. A comparison (later) with the cascade model singles out the decompression of excited nuclear matter as responsible for the observed collective motion in the reaction plane.

We now confront the findings with the theoretical models. From fig. 4 of ref. [7], showing the result of an ideal-fluid [4] calculation for ^{40}Ca + ^{40}Ca (0.4 GeV/nucleon) at b = 2 fm, we read off transverse momenta per nucleon in the forward and backward rapidity regions $\overline{\omega p^x/a}$ ~ 200 MeV/c. With hydrodynamic scaling [10] this would correspond to $\overline{\omega p^x/a}$ ~ 400 MeV/c at E_{lab} = 1.8 GeV/nucleon compared with the measured value of 95 ± 5 MeV/c. Though particle production should soften the rise of momenta with energy, it is clear that the momenta from fluid dynamics at 1.8 GeV/nucleon would be several times those observed experimentally.

From calculations of the Ar + KCl (1.8 GeV/nucleon) reaction at $b < 2.4$ fm, with the Cugnon [3] intranuclear code, we find for the nucleon momenta in the reaction plane $\overline{\omega p^x}$ = 22 ± 2 MeV/c, at $|y - y_B/2| > 0.3$. The average total transverse momentum of protons in the forward hemisphere is $\overline{p_f^x}$ = 0.23 ± 0.06 GeV/c. Since there may be some uncertainty in the impact parameters of the data, we quote also cascade results at fixed b = 3 fm: $\overline{\omega p^x}$ = 32 ± 3 MeV/c, $\overline{p_f^x}$ = 0.27 ± 0.05 GeV/c. Estimating with (4) we obtain $\overline{\omega p^x} \simeq$ 28 ± 7 MeV/c, and $\overline{\omega p^x} \simeq$ 31 ± 9 MeV/c, at $b < 2.4$ fm and b = 3 fm, respectively. We conclude that the cascade underestimates transverse momenta of the data by a factor of the order of four.

The current results do not include corrections for overall transverse momentum conservation. The corrections may be important when the moment analysis (3), (4), (10) is separately applied to the forward or backward hemispheres, for weaker dynamic effects than in the present data.

To conclude, we have successfully isolated collective motion effects in the transverse momenta from the Ar + KCl (1.8 GeV/nucleon) reaction. The apparent ease with which one gets a handle on the motion with this new method of analysis, stems from the fact that the method explores the two-particle correlation induced by existence of the reaction plane, amplified by summation over many particles. By contrast the standard sphericity method explores the single-particle [8] aspect of many particle distribution associated with the reaction plane. Even for low event statistics, eqs. (2)–(4) would allow for a distinction between trivial statistical fluctuations and dynamic effects, and permit a direct extraction of the magnitude of collective effects from the data. Some additional details of the analysis including extraction of elements of the sphericity matrix, are given in ref. [11].

We gratefully acknowledge the help from H. Pugh. We thank for the help and encouragement from all the other members of the LBL-GSI Streamer Chamber Group, in particular from J.W. Harris, R. Renfordt, H. Ströbele, and R. Stock. We want to thank M. Gyulassy for comments and stimulating discussions.

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