On the Hadronic Mass Spectrum.

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In the last few years an increasing number of hadron mass formulae and, recently. of speculations about the whole hadronic mass spectrum have been published—all of them based on group-theoretical considerations, quark models or the like.

We present here a different approach—a kind of asymptotic bootstrap—resulting from the « thermodynamical model » and dealing only with the spectral density $\varrho(m)$. The model has been described in three papers [1]: Statistical Thermodynamics of Strong Interactions at High Energies, I, II, III: the present consideration is a small but basic part of it.

In the thermodynamical model we describe highly excited hadronic matter by relativistic quantum-statistical thermodynamics, allowing arbitrary absorption and creation of hadrons (and antihadrons) of all kinds, including all resonances. As the spectrum of resonances cannot be limited, we take into account all of them, even the not yet discovered ones. It goes as follows: we introduce one common name: « fireballs » for all hadrons and postulate [the feed-back arrow is most important!]:

A fireball is

(T)   ┌──→ a statistical equilibrium of an undetermined
      │ number of all kinds of fireballs, each of which,
      │ in turn, is considered to be ──┐
      └─────────────────────────────────┘

We forget about complications like collective motions (in noncentral collisions) and imagine ideal equilibrium (realistic fireballs are discussed in (II)). One writes down the partition function $Z(V, T)$ for a gas consisting of an undetermined number of all kinds of particles (fireballs) which must be labelled, for instance, by their mass m. In calculating Z one has to sum over all single-particle momentum states, over all possible numbers of particles (bosons $0 \dots \infty$, fermions 0, 1) and over all possible *kinds of particles* (hadrons and antihadrons)—the latter is done by introducing the

[1] R. HAGEDORN: *Suppl. Nuovo Cimento*, **3**, 147 (1965) (I); R. HAGEDORN and J. RANFT: to be submitted to the *Suppl. Nuovo Cimento*, a preliminary report was given in CERN TH. 715, 30 September 1966 (II); R. HAGEDORN: to be published in *Suppl. Nuovo Cimento*, CERN preprint TH. 751 (III).

number of hadron states between m and $m + dm$: namely, $\varrho(m)\,dm$. With this (un-known) function $\varrho(m)$ the partition function becomes (see (I))

$$(1a) \qquad Z = \exp\left[\int_0^\infty \varrho(m)\,F(m,\,T)\,dm\right],$$

with a *known* function $F(m,\,T)$. On the other hand, Z can be written (see any book on statistical mechanics)

$$(1b) \qquad Z = \int_0^\infty \sigma(E) \exp\left[-E/T\right]dE,$$

where $\sigma(E)$ is the number of states between E and $E + dE$ of the fireball considered; as for this fireball $E = m$ (we stay in its rest frame) we can say as well that we have for our « main » fireball $\sigma(m)\,dm$ states in the mass interval $\{m, dm\}$. Now $\varrho(m)$ is the number of hadron states in the interval $\{m, dm\}$ and if our postulate (T) is applied it follows that asymptotically $\varrho(m)$ and $\sigma(m)$ must become somehow the same. A detailed discussion (see (I)) reveals that one cannot require more than that

$$(2) \qquad \frac{\log \varrho(m)}{\log \sigma(m)} \underset{m \to \infty}{\Rightarrow} 1,$$

which says that for $m \to \infty$ the entropy of a fireball is the same function of its mass, as is the entropy of the fireballs of which it is composed; this implies that *all fireballs are on an equal footing*.

We now equate the two expressions (1a) and (1b) and require simultaneously (2) to be valid. It is shown in (I) that $F(m,\,T)$ falls off asymptotically like $m^{\frac{3}{2}} \exp[-m/T]$ and that therefore

$$(1c) \qquad Z \Rightarrow \exp\left[\int_0^\infty m^{\frac{3}{2}} \varrho(m) \exp[-m/T]\,dm\right] \Leftrightarrow \int_0^\infty \sigma(m) \exp[-m/T]\,dm.$$

This is consistent with the bootstrap requirement (2) if and only if

$$(3) \qquad \varrho(m) \underset{m \to \infty}{\Rightarrow} \frac{\text{const}}{m^{\frac{3}{2}}} \exp[m/T_0] \quad (^*).$$

It follows that T_0 *is the highest possible temperature*—a kind of « boiling point of hadronic matter » in whose vicinity particle creation becomes so vehement that the temperature cannot increase anymore, no matter how much energy is fed in.

An immediate consequence is a Boltzmann-type momentum distribution (asymptotically $\sim \exp[-p_\perp/T]$) with $T \lesssim T_0$, but never larger than T_0! This explains why

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(*) It is not possible to have this $\varrho(m)$ cut off somewhere because this would imply two types of essentially different fireballs: one with almost exponential density of states, the other with asymptotically vanishing density of states and both would contribute and exist on an equal footing; this is inconsistent.

the transversal momentum distribution in high-energy jets is practically energy-independent (for all details and possible deviations see (II)).

Back to the mass spectrum: $\varrho(m)$ counts each state (spin, etc.) separately and includes antiparticles. If one smooths out the experimental mass spectrum [2], one obtains our figure in which an exponential increase is seen in the region $\leqslant 1000$ MeV, *i.e.* in that region where we know almost all resonances. Extrapolating the experimental curve with an expression having the required asymptotic behaviour (3) yields

(4) $$T_0 = (160 \pm 10) \text{ MeV}$$

and with this value excellent fits (ranging over 10 orders of magnitude) to the momentum spectra and multiplicities in high-energy production processes are obtained (see (II)). It is then only natural to expect [3] the form factors to decrease like $\sim \exp\left[-\sqrt{|t|}/4T_0\right]$.

We treat hadrons as self-consistently infinitely composed of all other hadrons—this is what (T) says. If all hadrons are virtually contained in each of them, it is natural to assume that all phase relations between the infinitely many contributing amplitudes wash out and that therefore statistical thermodynamics is adequate to treat this asymptotic bootstrap. Although the technique is unconventional, it is not so far from the usual ones as one might think; an intimate relation between the mass spectrum and the momentum distribution in multiparticle production seems unavoidable in any theory, and the Gibbs-ensemble description with fixed T somehow resembles the off-shell effects because the masses of fireballs present at temperature T extend to ∞ (with exponentially falling weight).

It will be impossible to prove or disprove our mass density (3) by direct experiments, because the density increases exponentially and the production cross-section for each individual resonance decreases exponentially with m—both mechanisms act in common against the experimenter.

Any « proof » of (3) will be indirect, but the internal consistency of the thermodynamic model and the good agreement of its predictions with an enormous amount of experimental data (see (II) and (III)) is a strong indirect support. In this respect, it is relevant that in the applications of the model ((II) and (III)) the asymptotic mass spectrum is used explicitly in integrals over m extending to ∞.

Recently, two papers [4] have been presented which use conventional quantum-mechanical techniques to construct infinitely composed, self-consistent hadrons. A variety of different model assumptions were shown to lead to one common behaviour: the form factors fall off asymptotically like $\exp\left[-\text{const }\sqrt{|t|}\right]$ in complete analogy with our result on momentum spectra. It seems then *that the sole requirement of self-consistent infinite composedness is sufficient to produce these asymptotically exponential laws for mass spectra, momentum distributions and form factors*—at least this is strongly suggested by the fact that the thermodynamical model does not make any other assumption and that in the papers by STACK and HARTE this assumption was the only one common to their various models.

In future, one should distinguish the « vicinity of the boiling point of hadronic

[1] A. H. ROSENFELD, A. BARBARO-GALTIERI, J. PODOLSKY, L. R. PRICE, P. SODING, C. G. WOHL, M. ROOS and W. J. WILLIS: *Rev. Mod. Phys.*, **39**, 1 (1967).

[2] T. T. WU and C. N. YANG: *Phys. Rev.*, **137**, B 708 (1965).

[4] J. STACK: *Rapidly decreasing form factors and infinitely composed particles*, University of Illinois preprint, June 1967; J. HARTE: *Crossing-symmetric bootstrap and exponentially falling form factors*, University of California preprint, August 1967.

matter » (*) where $T \to T_0$ and $E \to \infty$ and where literally all hadrons merge into each other. It follows from the small value $T_0 \approx 160 \, \text{MeV}$ (some $10^{12} \, °\text{K}$) that $E \to \infty$ means in this respect E above some $10 \, \text{GeV}$ (for quantitative relations, see (II)).

We conclude this letter with a curioisty—or perhaps not a curiosity. Consider a class of fireballs $f_n^{(i)}$ with roughly equal mass $m_n^{(i)}$, composed of quarks and antiquarks (altogether n of them (n large)). As the quark has 12 states ($SU_3 \times SU_2 \times$ antiparticle conjugation) this class of fireballs will have 12^n possible states ($i = 1, ..., 12^n$) if one assumes that each quark is in the ground state relative to all others (contrary to current models where, e.g., orbital momenta are discussed: here too they might be built in if one tries harder). Assume further that (as in nuclear physics) each of them contributes roughly the same and n-independent amount Δm to the average mass $\langle m \rangle_n$ of these fireballs, hence

$$(5) \qquad \langle m \rangle_n = \Delta m \cdot n \; .$$

The number of fireballs of mass $\approx \langle m \rangle_n$ becomes then for large n

$$(6) \qquad z(m) = 12^n = \exp[n \log 12] \, , \quad \text{or} \quad z(m) = \exp\left[\langle m \rangle \cdot \frac{\log 12}{\Delta m}\right] \; .$$

Δm can be estimated by using the meson 35-plet and/or the baryon 56-plet and taking the average mass of each of these multiplets:

$$(7) \qquad \Delta m = \frac{\langle m_{35} \rangle}{2} = \frac{\langle m_{56} \rangle}{3} \; ;$$

we find with $\langle m_{35} \rangle \simeq 700 \, \text{MeV}$ and $\langle m_{56} \rangle \simeq 1050 \, \text{MeV}$

$$(8) \qquad z(m) = \exp\left[\frac{\langle m \rangle}{140}\right], \quad \langle m \rangle \text{ in MeV} \; .$$
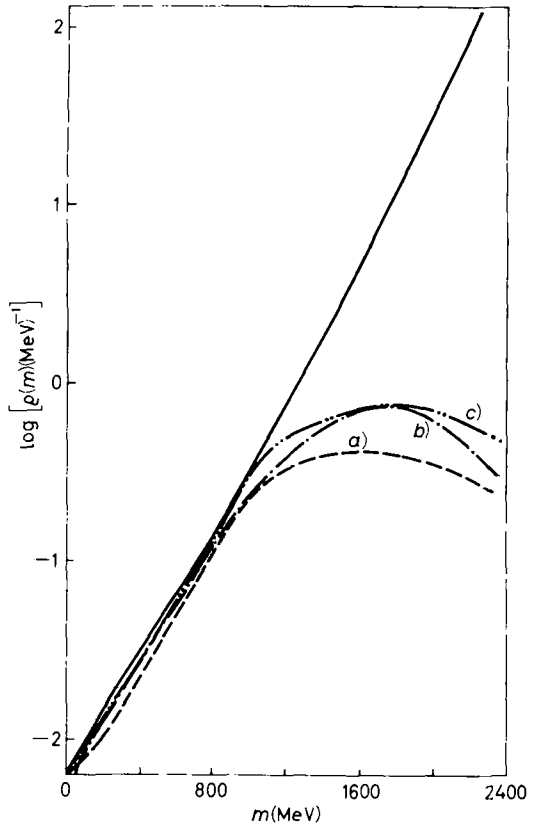
Fig. 1. – The experimental mass spectrum smoothed by Gauss functions (dashed lines) and a fit by a simple function with the asymptotic behaviour required by eq. (3). The constant a is a free parameter (with a value suggested by *a priori* considerations). $a = 2.53 \cdot 10^4 \, (\text{MeV})^{\frac{3}{2}}$, $m_0 = 500 \, \text{MeV}$, $T_0 = 160 \, \text{MeV}$. *a*) October 1964 (609 states); *b*) April 1966 (971 states), *c*) January 1967 (1432 states). A particle or resonance is counted with its statistical weight $z = (2j + 1)(2I + 1) \cdot 2^\alpha$ [$\alpha = 1$ if particle $\neq$ antiparticle, $\alpha = 0$ if particle $=$ antiparticle], and then represented by a Gauss function normalized to z with width 200 MeV.

(*) For symmetries, etc., one better looks at the « vicinity of the freezing point », so to speak; namely where most channels are frozen in.

It might be an accident that this is the leading term of (3) with a reasonable value of T_0. (It might be no accident.)

That this value is 140 MeV instead of 160 MeV as in the experimental spectrum causes $z(m)$ to grow faster than $\varrho(m)$—as it indeed should, because in our above class $\{f_n^{(i)}\}$ of fireballs objects with fractional charge and/or very large spin and isospin (up to $n/2$) are contained, which are not realized in the so far known experimental mass spectrum. Eliminating such states by means of supplementary conditions will, for large n, not spoil the exponential increase of the number of states, but make the slope smaller.

There is no contradiction in considering a fireball as built of fireballs and at the same time as built of quarks—superfluid helium is understood only if considered as a boson liquid, but after all it « really » consists of fermions: such pictures are complementary.