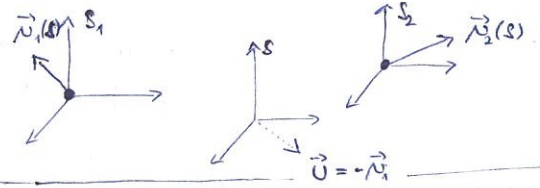
7.5 Relativní rychlost dvou částic je rychlost jedné částice v soustavě spojené s druhou částicí



stačí dosadit $\vec U = -\vec v_1$

$\vec v' = \vec v_2$

$$|\vec v| = \sqrt{\vec v\cdot\vec v} = \frac{1}{1+\frac{Uv'}{c^2}}\sqrt{\vec U^2 + \vec v'^2\left(1-\frac{U^2}{c^2}\right) + (1-\sqrt{\ })^2\frac{(\vec U\vec v')^2}{U^4}\vec U^2 + 2\vec U\vec v'\sqrt{1-\frac{U^2}{c^2}} + 2(1-\sqrt{\ })\frac{\vec U\vec v'}{U^2}U^2 + 2(\vec v'\vec U)\sqrt{\ }(1-\sqrt{\ })\frac{\vec U\vec v'}{U^2}} =$$

$$|\vec v| = \frac{1}{1+\frac{Uv'}{c^2}}\sqrt{\vec U^2+\vec v'^2+2\vec U\vec v' + \underbrace{\left(1-2\sqrt{\ }+\sqrt{\ }^2+2\sqrt{\ }-2\sqrt{\ }^2\right)}_{=\frac{U^2}{c^2}}\frac{(\vec U\vec v')^2}{U^2} - \frac{\vec v'^2\vec U^2}{c^2}} = \frac{1}{1+\frac{Uv'}{c^2}}\sqrt{(\vec U+\vec v')^2+\frac{(\vec U\vec v')^2}{c^2}-\frac{\vec U^2\vec v'^2}{c^2}} =$$

$U^2v'^2\cos^2\alpha - U^2v'^2 = -U^2v'^2\sin^2\alpha = |\vec U\times$

$$|\vec v| = \frac{1}{1+\frac{Uv'}{c^2}}\sqrt{(\vec U+\vec v')^2 - \left(\frac{\vec U\times\vec v'}{c}\right)^2}$$

tj. $$v_{rel} = \frac{\sqrt{(\vec v_2-\vec v_1)^2 - \frac{1}{c^2}(-\vec v_1\times\vec v_2)^2}}{1-\frac{\vec v_1\vec v_2}{c^2}}$$

7.6) Rapidita μ je definována pomocí vztahu $\operatorname{tgh}\mu = \frac{v}{c} = \beta$

Ze vzorce př. 7.5 pro relativní rychlost odvoďte "kosinovou větu" pro rapiditu

$$\operatorname{tgh}\mu = \frac{\sinh\mu}{\cosh\mu} \qquad 1-\operatorname{tgh}^2\mu = \frac{\overbrace{-\sinh^2\mu+\cosh^2\mu}^{1}}{\cosh^2\mu}$$

$$\cosh^2\mu = \frac{1}{1-\operatorname{tgh}^2\mu} \Rightarrow \cosh\mu = \frac{1}{\sqrt{1-\beta^2}} = \gamma$$

$$\sinh\mu = \sqrt{\cosh^2\mu - 1} = \beta\cdot\gamma$$

$$\frac{1}{\sqrt{\ }^2} - 1 = \frac{1-1+\beta^2}{1-\beta^2} = \beta^2\gamma^2$$

$$1-\frac{v_{rel}^2}{c^2} = \frac{1}{\left(1-\frac{\vec v_1\vec v_2}{c^2}\right)^2}\left[1 - \frac{2\vec v_1\vec v_2}{c^2} + \frac{(\vec v_1\vec v_2)^2}{c^4} - \frac{v_2^2}{c^2} + \frac{2\vec v_1\vec v_2}{c^2} - \frac{\vec v_1^2}{c^2} + \underbrace{\frac{1}{c^4}(\vec v_1\times\vec v_2)^2}_{-(\vec v_1\vec v_2)^2+\vec v_1^2\vec v_2^2}\right]$$

$$= \frac{\left(1-\frac{v_1^2}{c^2}\right)\left(1-\frac{v_2^2}{c^2}\right)}{\left(1-\frac{\vec v_1\vec v_2}{c^2}\right)^2} = \frac{1}{\gamma_{rel}^2} = \frac{1}{\cosh^2\mu_{rel}} \Rightarrow \cosh\mu_{rel} = \cosh\mu_1\cdot\cosh\mu_2\cdot\left(1-\frac{\vec v_1\vec v_2}{c^2}\right) = \cosh\mu_1\cosh\mu_2 - \sinh\mu_1\sinh\mu_2\frac{\vec v_1\vec v_2}{|\vec v_1||\vec v_2|}$$